An Extended Tropospheric Scintillation Model for Free Space Optical Communication Systems

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Abstract
Fluctuations caused mostly by tropospheric scintillation at the free space optical receiver end have been a major problem in the rapid development of telecommunication and the increasing demands for larger bandwidth is forcing the use of free space optical (FSO) technology. This paper examined existing tropospheric scintillation models of Karasawa, Van de Kamp model, Otung, Ortgies and ITU-R, and discovered that all of them operate at the microwave range, which limits their application in FSO laser beam technology that operates in PHz frequency-range. ITU-R model was later selected owing to its global application and modified for use in FSO communication system. The new model can serve as basis for communication engineers to use as platform in the link budgetary for planning and design of low margin systems of free space optical communication link.

Keywords: Signal fluctuation, Tropospheric Scintillation Model, FSO, ITU-R

1. Introduction
The uptake of free-space optical (FSO) transmission is constrained by attenuation due to fog and cloud as well as scintillation fading. Scintillation is the random optical-power fluctuations in signal amplitude, which leads to image fluctuations at the FSO receiver end caused by atmospheric turbulence as a result of refractive index variation [1]. In the assessment of a link budget, it is of great importance to quantify the losses caused by scintillation in terms of power loss. This is to account for additional power that will be needed to overcome scintillation effects and thus to reach the required performance. Scintillation loss is a major issue to address in optical wireless communication system.

Scintillation is categorized into two types: ionospheric scintillation, and tropospheric scintillation. Ionospheric scintillation is a rapid fluctuation of radio-frequency signal amplitude and/or phase, generated as a signal traverses through the ionosphere causing small-scale irregularities in electron density. The presence of these charged particles makes the ionosphere an electrical conductor, which supports electric currents and affects radio waves [2], thereby affecting trans-ionospheric radio signals of frequencies up to 6 GHz. Tropospheric scintillation is a rapid fluctuation of signal amplitude and/or phase due to turbulent irregularities in temperature, humidity, and pressure, which translate into small-scale variations in refractive index [3]. In the tropics where this research study is conducted, as well as in the equatorial regions, the humidity fluctuations are important because they cause random degradation and enhancement in signal amplitude and phase received on a satellite–earth link, as well as degradation in performance of large antennas. Moreover, on the line of sight linkup through 10 GHz and on earth-space paths at frequencies above 50 GHz, the tropospheric scintillation is often detected [4].

This research work focuses on the tropospheric scintillation; as such, what constitutes a troposphere is described as follows. The troposphere is the lowest and unstable layer of Earth's atmosphere, where most of the weather phenomena, systems, convection, turbulence and clouds occur. The troposphere contains 99% of the water vapour-whose concentrations vary with latitudinal position-in the atmosphere [2]. The height of the troposphere varies with location being higher over warmer areas and lower over colder areas; it ranges between 10km-12km [5].

Received July 3, 2014; Revised August 28, 2014; Accepted September 20, 2014
Tropospheric scintillation occurs with or without rain (clear air or sky); it is therefore a serious concern in free space optical channel which impairs the availability and reliability of the system. However, clear air turbulence has long been identified as a primary source of scintillation. Models that focus on clear air effects, as well as fair-weather cumulus clouds crossing the transmission link, include those of [3], [6], [7], [8]. Research studies continue, though, in predicting tropospheric scintillation both theoretical and empirical. Regardless of the methods employed, inclusion of the main link parameters (for example, the frequency, elevation angle and antenna diameter) and meteorological data (for instance, the humidity at ground level and mean temperature) are needed in order to obtain reliable scintillation prediction. Tropospheric scintillation is therefore a signal propagation impediment, which must be accounted for in order to complete the link budget for design of low margin systems.

2. Tropospheric Scintillation Models

There are many prediction models that have been proposed over the years in order to estimate the statistical distributions of scintillation. The required input parameters needed for these models are signal frequency $f$ (GHz), antenna diameter $D$ (m), path elevation angle $\Theta$ (deg), average temperature ($^\circ$C), and average relative humidity (%) which are readily available. However, all the proposed tropospheric scintillation models are usable and applicable for signals frequencies in the GHz range because of the higher wavelength (microwave) as compared to low wavelength in which FSO system operates. The losses are more pronounced on FSO communication systems, which should be accounted for. Some of the current tropospheric scintillation models are presented in the following subsections.

2.1. Karasawa Scintillation Prediction Model

This is a measurement based prediction model made in the year 1983, Yamaguchi city of Japan at an elevation angle of $6.5^\circ$, frequencies of 11.5 and 14.23 GHz and an antenna diameter of 7.6m [1]. The following prediction formulae were derived using data:

$$\sigma_{pre} = 0.0228(0.15 + 5.2 \times 10^{-3} N_{wet}) dB f^{0.45} \sqrt{G(G_e)/sin^{1.3}\varepsilon}$$  \hspace{1cm} (1)

where:

- $\sigma_{pre}$ = The predicted signal standard deviation or scintillation intensity
- $f$ = Frequency (GHz)
- $\varepsilon$ = Apparent elevation angle (degree)
- $G(G_e) = $ Antenna averaging
- $G_e = $ Effective antenna diameter given by:

$$D_e = D \sqrt{\eta}$$  \hspace{1cm} (2)

- $D = $ Geometrical antenna diameter (m)
- $\eta = $ Antenna aperture efficiency

This prediction model indicated that the antenna averaging function also depends on the elevation angle and the height of the turbulence to be 2000m. If $\varepsilon < 5^\circ$, $\sin \varepsilon$ in Eqn. (1) should be replaced by:

$$\sin \varepsilon + \sqrt{(\sin^2 \varepsilon + \frac{2h}{R_e})/2}$$  \hspace{1cm} (3)

where,

- $h = $ Height of the turbulence (m)
- $R_e = $ Effective earth radius = $8.5 \times 10^8$ m [6]

The effective earth radius varies with latitude, i.e. as one move away from the equator. Nigeria location is above the equation with $R_e$ of 6378km, which is fractionally different from that of Japan which Karasawa quoted.

The following equation is the wet term of the refractivity at ground level:
\[
N_{\text{wet}} = \frac{22790 U e^{(19.7 t + 273)}}{(t + 273)^2} \text{ (ppm)}
\] (4)

where

\( N_{\text{wet}} = \) Relative humidity (%) due to water vapor in the atmosphere

\( t = \) Temperature (°C)

\( U = \) Relative humidity (%)

Karasawa et al (1988) also presented that the meteorological input parameters should be averaged over a period in the order of a month so the model does not predict short-term scintillation variations with daily weather changes. The equations for the scintillation enhancement \( (n(p^+)) \) and scintillation fading \( (n(p^-)) \) are respectively expressed as follows:

\[
n(p^+) = -0.0597 \left( \log(100 - p) \right)^3 - 0.0835 \left( \log(100 - p) \right)^2 - 1.258 \left( \log(100 - p) \right) + 2.672, \text{ for } 50 < p \leq 99.99
\] (5)

\[
n(p^-) = -0.061 \left( \log p \right)^3 + 0.072 \left( \log p \right)^2 - 1.71 \left( \log p \right) + 3.0, \text{ for } 0.01 < p \leq 50
\] (6)

To determine the cumulative time distribution for the scintillation enhancement \( (X(p)) \) and scintillation fade \( (\sigma_{\text{pre}}) \) has to be included in Eqns. (5) and (6); specifically,

\[
X(p) = n(p^+) \times \sigma_{\text{pre}}
\] (7)

\[
X(p) = n(p^-) \times \sigma_{\text{pre}}
\] (8)

Generally, it could be observed that the model approach was on Intelsat and applicable to wide regions under different climate most especially where the research was carried out. However, the data used here are for four seasons (namely: Winter, Autumn, Summer and Spring) only and does not include desert or tropical region.

### 2.2. Ortgies Scintillation Prediction Models

Ortgies (1993) presented two models: Ortgie-Refractivity (Ortgie-R) and Ortgie-Temperature (Ortgie-T). The experiment was conducted on Olympus satellite measurements at Darmstadt, Germany. The frequencies used were 12.5, 20 and 30 GHz. Ortgies applied a log-normal probability density function (pdf) for long term distribution of scintillation intensity parameters; \( \mu \) and \( \sigma \) which are mean and standard deviation of \( \ln(\sigma^2) \) respectively [9]. The two models are based on direct proportional relationships that exist between mean surface measurement and monthly mean normalized log variance of scintillation. Ortgies-T model takes the monthly mean surface temperature \( (T) \) as a predictor:

\[
\ln(\sigma_{\text{pre}}^2) = \ln[(g^2(x)k^{1.21}(\sin\theta)^{-2.4} \times 12.5 + 0.0865(T)]
\] (9)

Whereas the Ortgies-N model uses monthly mean log-variance of signal log-amplitude to monthly mean wet component of surface refractivity \( (N_{\text{wet}}) \) as a predictor:

\[
\ln(\sigma_{\text{pre}}^2) = \ln[(g^2(x)k^{1.21}(\sin\theta)^{-2.4}] - 13.45 + 0.0462(N_{\text{wet}})
\] (10)

However, the models are not appropriate for tropical or desert climate, though it includes meteorological parameters, \( N_{\text{wet}} \).

### 2.3. Otung Scintillation Prediction Model

Otung [10] worked on the prediction of tropospheric amplitude scintillation. A simple expression was proposed for the annual and worst-month cumulative distributions of scintillation fades \( x^- \) and enhancements \( x^+ \) which are applicable to predict scintillation on a satellite link. The scintillation data were obtained at Sparshot, UK (51.5850N, 1.5033W) for a period of one year by the use of Olympus satellite 19.7704 GHz beacon observed at elevation angle 28.74°. This model is related to the ITU-R model except a little modification in the elevation angle of the scintillation fade, expressed as:
\[
\sigma_{pre} = \frac{\sigma_{pre} f_{\Omega}(x)}{(\sin \theta)^{11/12}}
\]  

(11)

For annual distribution, the scintillation fades, \(x_a\), and scintillation enhancement, \(x_{+a}\), are written as:

\[
x_a = 3.6191\sigma_{pre} \exp \left( -\frac{9.50142 \times 10^{-4}}{p} \left[ 0.40454 + 0.00285p \right] \ln(p) \right)
\]  

(12)

\[
x_{+a} = 3.1782\sigma_{pre} \exp(0.0359654p - [0.272113 - 0.00438] \ln(p))
\]  

(13)

where \(a\) is the annual distribution, and \(p\) is time percentage factor. For worst-month distribution, the scintillation enhancement and scintillation fade \(x_w\) and \(x_{-w}\) respectively, are written as:

\[
x_w = 6.8224\sigma_{pre} \exp \left( -10^{-4} \frac{9.1312}{p} + 1.8264p^2 \right) - \frac{0.023027}{p} + 0.51664 \ln(p)
\]  

(14)

\[
x_{-w} = 5.5499\sigma_{pre} \exp \left( -10^{-4}[946.849p + 4.4974p^2] + [0.02357p - 0.261135] \ln(p) \right)
\]  

(15)

Whilst the Otung (1996) model provides worst-month and annual distributions of scintillation, it is not applicable to tropical climate conditions.

2.4. Van De Kamp Tropospheric Scintillation Model

Van de Kamp et al. [3] deployed the ITU-R model in their prediction model but a small change in the elevation angle as in Eqn. (16). This model was derived and tested in four sites in different climates: Japan, United Kingdom, Finland, and Texas by scintillation measurements. Van de Kamp et al. [3] model introduced the cloud type information based on edited synoptic cloud reports, which observed that there was a scintillation correlation between the occurrence of scintillation and the presence of cumulus clouds. Also, Mayer [11] published an improved version of the Van de Kamp et al. [3] model, and that heavy clouds are clouds with integrated water content larger than 0.7 kg/m². He incorporated \(W_{hc}\) into the model, thus:

\[
\sigma_p = \frac{0.45}{\sin^{1.3}(e)} \sqrt{g^2(D_e)} 0.98 \times 10^{-3}(N_{wet} + Q)
\]  

(16)

\[
Q = -39.2 + (W_{hc})Q
\]  

(17)

where

- \(W_{hc}\) = Average water content of heavy clouds [kg/m²]
- \(x\) = Long-term (at least) average of the parameter \(x\)
- \(Q\) = Long-term average parameter and hence constant for each site, so that all seasonal dependence of \(\sigma_p\) is still represented by \(N_{wet}\)

Van de Kamp et al (1999) also adopted formulae for scintillation enhancement and scintillation fade depth. Specifically,

\[
a_1(p) = -0.0515(\log_{10}p)^3 + 0.206(\log_{10}p)^2 - 1.5 - 81\log_{10}p + 2.18
\]  

(18)

\[
a_2(p) = -0.172(\log_{10}p)^2 - 0.454\log_{10}p + 0.274
\]  

(19)

where

- \(a_1(p)\) and \(a_2(p)\) are time percentage factors:

\[
E_p(p) = a_1(p)\sigma_p - a_2(p)\sigma_x^2 \text{ for } 0.001 \leq p \leq 0.20
\]  

(20)
$a_2(p) = a_1(p)\sigma_p + a_2(p)\sigma_y^2 \text{ for } 0.001 \leq p \leq 20$ \hspace{1cm} (21)

where $E_p(p)$ and $a_2(p)$ are scintillation enhancement and scintillation fade depth, respectively. It was observed that the scintillation enhancement and scintillation fade depth in Van de Kamp et al\[3\] model are meant for the percentage factors from 0.001 till 20, but this is in contrast to Karasawa et al\[6\], Otung\[10\] and ITU-R\[7\] models whose percentage factor is between 0.001 and 50. Van de Kamp scintillation prediction model includes cloud information and has significant improvement on the accuracy of scintillation variance. However, the model is on experimental data from limited sites, may be as a result of scarcity of experimental data and cannot be used for tropical climatic condition.

2.5. ITU-R Tropospheric Scintillation Model

A tropospheric scintillation model was developed by international telecommunication union of radio section (ITU-R), which has frequencies between 7 - 14 GHz and theoretical frequency dependence and aperture averaging effects, estimates the average scintillation intensity $\sigma_{per}$ over a minimum period of one month\[1\]. The input parameters required for this model are: signal frequency $f$ (GHz), antenna diameter D (m), path elevation angle $\theta$, average temperature ($^\circ$C) and average relative humidity $U(\%)$ which are readily available. The elevations angle used for the model is between 4 0 and 320 and the antenna diameters used is between 3 and 36m. Also in the ITU-R scintillation model, the long term scintillation variance is expressed as a relationship with $N_{wet}$; which is a function of relative humidity $U(\%)$ and temperature t ($^\circ$C), measured at ground level (P. 618-10 2009):

$$N_{wet} = 3.732 \times 10^5 \frac{e}{T^2} \hspace{1cm} (22)$$

For the temperature range of -20 to 50$^\circ$C, the ITU-R P453-9 defined the water vapour pressure as:

$$e = 0.01 \times U \times \left(6.1121 \exp \left[\frac{17.502}{t+248.69}\right]\right) \hspace{1cm} (23)$$

where
e: water vapour pressure (hPa)
T: absolute temperature (K)
t: Celsius temperature ($^\circ$C)
U: relative humidity (\%)
The standard deviation of the signal fluctuation due to scintillation is given by:

$$\sigma = \sigma_{ref} f^{\frac{7}{12}} \left[\frac{g(x)}{(\sin \theta)^{\frac{11}{12}}}\right] (dB) \hspace{1cm} (24)$$

where

$\sigma_{ref} =$ Normalized or reference standard deviation given by:

$$\sigma_{ref} = 3.6 \times 10^{-3} + N_{wet} \hspace{1cm} (dB) \hspace{1cm} (25)$$

g(x) = \text{Antenna averaging factor}

$$g(x) = \sqrt{\frac{3.86(x^2 + 1)^{\frac{11}{12}} \sin \left(\frac{11}{6} \arctan \frac{1}{x}\right) - 7.80x^{\frac{5}{8}}}{x}} \hspace{1cm} (26)$$

where

$$x = 1.22D_{eff}^2 \left(\frac{f}{L}\right) \hspace{1cm} (27)$$

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Effective diameter

\[ D_{\text{eff}} = \sqrt{\eta D} \]  

(28)

\( D \) is antenna aperture diameter; \( \eta \) is antenna efficiency (0 ≤ \( \eta \) ≤ 1).

\[ L = \frac{2h_L}{\sqrt{(\sin \theta)^2 + 2.35 \times 10^{-4} + \sin \theta}} \quad (m) \]  

(29)

where:

\( h_L \) = Height of the turbulent layer; \( h_L = 1000 \text{m} \)

\( \theta \) = Elevation angle

This model is applicable to wide regions of different climates. However, it cannot be used in tropical regions as well as in an atmosphere that is dry. It has an advantage of being applicable everywhere. Other tropospheric scintillation models (most especially those discussed above) are modifications of ITU-R tropospheric scintillation model. However, all the proposed tropospheric scintillation models, including ITU-R model are usable and applicable for signals frequencies in the GHz range. So, because of the higher wavelength, microwave possesses compared to low wavelength in which FSO system operates, this may introduce higher absorption due to rainfall, scattering, reflection, refraction and fading, which in turn increases the unavailability and unreliability of the free space optical communication systems. This rendered these models less applicable to FSO technology. Therefore, there is a need to propose a scintillation model that will fit in for FSO systems (Laser beam) which operates in PHz frequency range. This FSO scintillation model will account for the higher fluctuation of the amplitude and phase of the beam signal at the receiver end, so that wireless communication engineers can have a better platform to work with in order to reach their target, which is optimal performance by providing a reliable network and high quality of service.

3. Research Method

ITU-R tropospheric scintillation model—as stated in [1], i.e. Eqn. (22)—was used as a slave model to determine a scintillation model that fits for FSO spectrum. The model in its original form has its application in microwave (GHz frequency range) but FSO laser or beam signal operates in PHz (0.1 to 10PHz) frequency range. The two cases are considered: ITU-R model with microwave mean frequency; and ITU-R model with FSO (laser beam) mean frequency range. The two results were added and averaged to determine the suitable tropospheric scintillation model for FSO communication systems.

All the parameters are as defined in Sec. 2.2.5. The efficiency is assumed to be unity though this may not be so in practice. Turbulent height (\( h_L \)) was taken to be 1000m as proposed by ITU-R. Elevation angle \( \Theta \) under consideration is 30\(^\circ\), which is within the range of ITU-R. The temperature \( t \) and relative humidity \( H \) were set at 37.1\(^\circ\)C and 24\% respectively and antenna aperture diameter was taken to be 15m. Matlab Simulink Software Package was used for the simulation of the model under the two cases and plot of the standard deviation \( \sigma \) against frequency \( f \) for both microwave mean and laser mean frequencies were generated, also the average of the two cases was determined using the same Matlab software package.

4. Results and Discussion

Equation (24) is the slave model considered for the following two cases:

Case 1: Standard deviation \( \sigma \) at microwave mean frequency spectrum (12 to 20 GHz) is denoted as \( \sigma_1 \), i.e.

\[ \sigma_1 = \sigma_{\text{ref}}\sqrt{\frac{g_1(x)}{(\sin \theta)^{1/2}}} \quad (dB) \]  

(30)

The simulation result is presented in Figure 1.
Case 2: Standard deviation $\sigma$ at laser mean frequency spectrum (0.1 to 10 PHz) is denoted as $\sigma_2$. 

$$\sigma_2 = \sigma_{\text{ref}} f_2^{\frac{7}{12}} \left[ \frac{g_2(x)}{(\sin \theta)^{1.2}} \right] (dB) \quad (31)$$

The simulation result is presented in Figure 2. The two cases were averaged to give: $\frac{\sigma_1 + \sigma_2}{2}$ (i.e. averaging Eqns. (29) and (30)):

$$\frac{\sigma_{\text{ref}}}{2(\sin \theta)^{1.2}} \left[ f_1^\frac{7}{12} g_1(x) + f_2^\frac{7}{12} g_2(x) \right]$$

$$\quad (32)$$

It is justifiable to say, that since $f_1 << f_2$ and $g_1(x) << g_2(x)$ then, it is ascertained to say that $f_1^\frac{7}{12} g_1(x) << f_2^\frac{7}{12} g_2(x)$, and consequentially, if the above holds, we formulate a new tropospheric scintillation model suitable for free space optical communication system as:

$$\sigma = \frac{\sigma_{\text{ref}}}{2(\sin \theta)^{1.2}} \left[ f_1^\frac{7}{12} g_1(x) \right]$$

$$\quad (33)$$

All the parameters are as defined by ITU-R except the frequency range which is now 1 to 5.5 PHz as evident in Figure 3.
5. Conclusion

The uptake of free-space optical (FSO) transmission is constrained, among other things, by scintillation fading. Tropospheric scintillation is predominated in the tropics, which this paper investigated. This paper has presented a modified ITU-R tropospheric scintillation model.
that is usable at laser beam frequency spectrum (free space optical communication system). It provides basis for communication engineers to use as platform in the link budgetary for planning and design of low margin systems of free space optical communication link.

References


