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SISO System Model Reduction and Digital Controller Design using Nature Inspired Heuristic Optimisation Algorithms

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ABSTRACT

This research article explores an algorithm to reduce the order of a SISO system and thereby to design a digital controller. The reduced order modelling of a large complex system eases out the analysis of the system. AGTM (Approximate Generalised Time Moments) method was implemented wherein the responses were matched at different time instants to achieve the reduced system. This research work devises a new method, Ensemble Framework for Optimized System (EFOS), resulting into a reduced system with better performance as compared to conventional techniques. The research also efforts towards effective utilization of various heuristic algorithms like Genetic Algorithm, Particle Swarm Optimization and Luus Jaakola Algorithm, their implementation and a comparison with other techniques based on relative mean square error and time complexity. It was observed that the proposed transfer learning based approach , EFOS, combining the advantages of Luus Jaakola and Genetic algorithms depicted better results than their individual counterparts on diverse performance parameters like speed of convergence and optimal convergence to global minima. The percentage improvement achieved in the time taken for design of the digital controller was 85.3%, with no change in delta value

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1. INTRODUCTION

The advent of digital domain and digital computers as system controllers, superseded many traditional analog controllers in various fields of Engineering. A Digital control system is analogous to a micro-controller to an ASIC to a traditional computer and uses a discrete system, making usage of the Z-transform instead of Laplace transform. The major advantage provided by a digital control system is the viability to implement complex control laws, resolving the non-linearities, construction tolerances or parameter variations via auto-tuning strategies and self- analysis, which is very difficult to implement analogically. The flexibility which the digital controller offer, in turn allows the flexibility to modify the control strategy, or to entirely reprogram it, without the need for significant hardware modifications. The critical parameters for design of control system are the higher tolerance to signal noise and the complete absence of thermal drifts or ageing effects [1]. The advantages of being inexpensive and scalable, makes them more suitable for implementations in case of static operations as they are much less bulky and less prone to environmental conditions than analog counterparts like capacitors & inductors.

Physical systems modelling usually results in high order and complex dynamic formulaes. Thus, design and simulation of the higher order systems controller is mostly a complex problem statement. The complexity and the cost involved with the controller increases with system's order. It is necessary to approximate these complex models to model with lesser order, as lower order models preserves, reflects and represents all salient features of higher order models [2]. Modern controller design methods, as 'Model

Matching Technique' is used for designing the higher order controller. It is then approximated to a lower order model by application of Approximate Generalized Time Moments (AGTM) / Approximate Generalized Markov Moments (AGMM) matching technique. Furthermore, Genetic Algorithm (GA) optimization technique is used to derive the expansion points which yields similar response as that of model, thus minimizing error between the response of the model and that of designed closed loop system.

The research fraternity of this domain is looking into the various parameters to design the model with lower order. The research work in this paper focusses on idea of developing a digital controller that can be used in various real time applications for both SISO and MIMO systems. The performance of the designed model is also checked by observing the results forr system reduction of controller for discrete time invariant systems and for Linear time invariant Discrete time systems. Various evolutionary algorithms inspired by nature and complex processes involving certain natural phenomenon are considered for the research work. The proposed design model is evaluated in terms of standard performance metrics as MSE, Generations, time complexity and compared with state of the art research, in subsequent sections. Algorithms employed for designing the models and reducing the order of the model in this research are:

- Genetic Algorithm
- Particle Swarm Optimization
- Differential Evolution
- Luus Jaakola Algorithm for Optimisation

The qualitative inspiration behind employing various heuristic algorithms have been derived from different phenomenon of nature and natural processes, and the quantitative benefit is decrease in the time complexity of problems by giving quick solutions.

A. Genetic Algorithm

Genetic Algorithm (GA) replicates Darwin's theory of evo- lution *Survival Of Fittest*. The algorithm consists of various input parameters depending on the objective function. The input parameters resembles the variables used by Darwin in his evolution theory [3]. The parameters are encoded in *chromosomes* (form of strings) and a collection of such strings is called a *'Population'*. A random population is created initially, which further identifies several points in the search space.

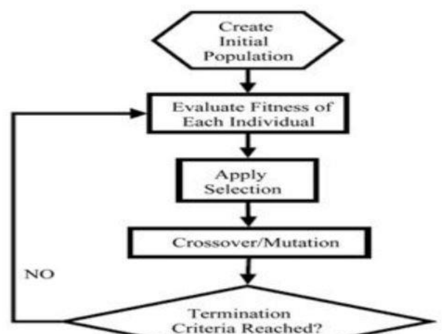


Figure 1. Genetic Algorithm flow chart

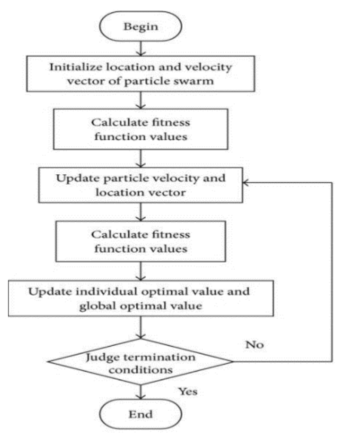


Figure 2. PSO Algorithm flow chart

An objective and a fitness function is related with each of the string which in turn represents the degree of goodness of that string. Based on the algorithm resembling, Survival of the fittest, few strings are selected and each of the string is allocated, a particular number of copies that gets included into the mating pool [4]. Operators inspired from biology, like Cross-Over and Mutation are applied on these strings to form a new gen of strings. GA's perform a search in this large, complex and multi modal landscapes, and provide close to optimal solutions for objective / fitness function of an optimization problem [5]. Figure 1 provides the flow diagram of the algorithm. Each of the block is mathematically modelled either with real coded GA or with binary coded GA.This algorithm terminates when the optimal value is reached in between the generations.

B. Particle Swarm Optimization

The algorithm consists of various input parameters depending on the objective function on which the convergence depends. A PSO swarm member/ agent (a particle) continuously (in an iterative manner) modifies a complete solution [7]. It requires merely primitive mathematical operators, and is computationally inexpensive in terms of both speed and memory requirements [8]. Figure 2 gives the flow diagram of PSO algorithm. PSO algorithm comprises of swarm of bird-like particles. Each of the particle has a specific loci in the search space. The fitness of the each particle, signifies the quality of its position. These particles move over the search space with a specific velocity. Each particle has an internal state and a network of social connections. The velocity (both speed and direction) of each particle is influenced by its own best position found so far, *pbest*, the best solution that was found so far by its social neighbors, *lbest*, and/or the global best so far *gbest*. "Eventually" the swarm will converge to optimal positions. [9].

C. Differential Evolution

This algorithm works similar to GA but the offspring's are determined by the difference of the parents. When the objective function of differential evolution is nonlinear and non-differentiable, direct search approaches are utilised. The basic strategy of DE algorithm employs the difference of two randomly selected parameter vectors as the source of random variations for a third parameter vector [9]. Figure 3 and 4 presents simple flow chart of DE algo and flow of algorithm employed in this research article respectively

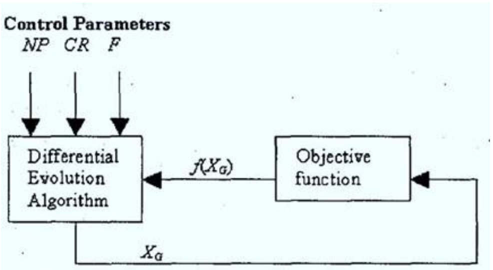


Figure 3. Differential Evolution flow chart [10]

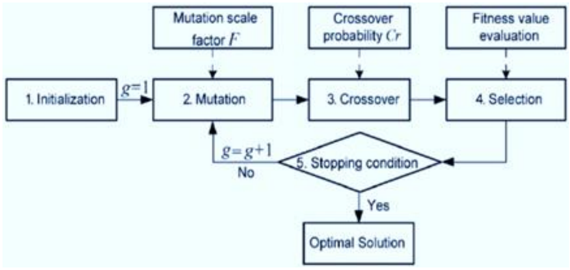


Figure 4. Differential Evolution flow diagram [11]

D. Luus Jaakola Optimisation

Luus Jaakola optimization procedure is used effectively for optimization of complex systems as heat exchanger networks, and it has been employed optimally for the minimization ofthe Gibbs free energy for single phase situations [6]. In Luus Jaakola optimization algorithm, initial test points over some defined region are chosen in an arbitrary fashion. The sizeof this region then contracts in further iterations with thebest values derived in previous iterations. This is the fastest optimisation algorithm which converges the coefficient valuesrapidly in less time.The algorithm flow is defined in Algorithm1.

Algorithm 1 Luus Jaakola Optimisation Algorithm

1: Initialize x = Set of possible (arbitrary values) coefficient values of reduced model.
U($bdry_{up}, bdry_{lo}$) with a random uniform position in the search-space, where $bdry_{lo}$ and $bdry_{up}$ are the lower and upper boundaries, respectively.

2: Set the initial sampling range d to cover the entire search-space (or a part of it):

$$d = bdry_{up} - bdry_{lo} \quad (1)$$

3: Until a termination criterion is met, (e.g. number of iterations performed, or adequate fitness reached), repeat the following:

- Pick a random vector a U(- d, d)
- Add this to the current position x to create the new potential position

$$y = x + a \quad (2)$$

- Add this to the current position x to create the new potential position y = x + a
- If (f(y) > f(x)) then move to the new position by setting x = y, otherwise decrease the sampling range: d = 0.95 d.

4: Now x holds the best-found position

This article provides an exhaustive comparative analysis of various algorithms for system reduction that are inspired by various Heuristic Optimization Algorithms. The algorithms used in this research for system reduction are Genetic Algorithm, Particle Swarm Optimization, Differential Evolution and Luus Jaakola Algorithm for Optimization. This article further proposes a Heuristic Optimization Algorithm inspired model reduction approach of SISO System. It further devises a new method, Ensemble Framework for Optimized System (EFOS) which yields a reduced system with better performance as compared to conventional techniques. The research also focuses on effective utilization of various heuristic algorithms like Genetic Algorithm, Particle Swarm Optimization and Luus Jaakola Algorithm, their implementation and a comparison with other techniques is done based on relative mean square error and time complexity. This research work reveals that the proposed transfer learning based approach , EFOS, combining the advantages of Luus Jaakola and Genetic algorithms depicted better results than their individual counterparts on diverse performance parameters like speed of convergence and optimal convergence to global minima.

The rest of this article is organized as: section 2 provides the literature review, however, section 3 depicts the research method describing the model reduction methodology. Section 4 presents the results and discussion, followed by the concluding remarks in section 5.

2. LITERATURE REVIEW

There are several methods reported in the literature which uses the optimization technique for reducing the order of the system. Some of these approaches are based on error minimization between the original and the reduced system [12]. The errors are considered in the current literature in terms of integral absolution error [13], integral square error [14,15] or several weighted error functions [16]. Some of the meta-heuristic approaches are also considered by various researchers which are nature inspired optimization approaches [17]. Genetic algorithms [18] is one of the most widely used optimization approaches which works on the criteria of survival of the fittest. There are several metaheuristic approaches which are based on foraging conduct of population [19] like particle swarm optimization (PSO), Harmony Search Algorithm (HSA) and Cuckoo Search Algorithm (CSA). The CSA algorithm is centered on the behavior of flocking birds [20], however, HSA takes into account the improvisation of musical instrument [21]. The CSA algorithm considers the laying of eggs by cuckoo bird in other host bird's nest, lays the basic foundation of this algorithm [22,23]. Some of these approaches have successfully been applied to the various fields like parameter identification in electrical systems, order reduction and adaptive control optimization [24,25].

These approaches are further being used for better optimization and to obtain better performance in various research fields. Authors in the previous literatures have used several classical optimization approaches in order to reduce the order of a control system. Authors in [26] have proposed a single-input single-output (SISO) and multi-input multi-output (MIMO) methodology for depicting the dominant poles in the original system. They have selected various dominant poles for parameter optimization using PSO algorithm. The authors in [27] have proposed a Big Bang–Big Crunch optimization methodology for estimation and approximation based on Routh criteria. They have also utilized the approach of time moment matching. Several other researchers in [28] utilized the hybrid metaheuristic approaches for system time-discretization using the combination of Gray Wolf optimization and Firefly algorithm.

Despite of several research initiatives in this field, the system approximation methods are still unexplored for several problem specific applications. The research gaps in the present approximation methods are their large error percentage in resultant system. Moreover, these approaches are computationally complex as well as their time complexity is also very high. These gaps motivated this study for exploring a new and much effective approach which is computationally less complex.

This article presents a new Heuristic Optimization Algorithm inspired model reduction approach of SISO System. The proposed Ensemble Framework for Optimized System (EFOS) yields a reduced system with better performance as compared to conventional techniques. The presented method minimizes the relative mean square error and reduces time complexity. The combined approach EFOS takes the benefits of Luus Jaakola and Genetic algorithms providing better performance in terms of speed of convergence and optimal convergence to global minima.

3. RESEARCH METHODOLOGY

3.1. Model Reduction For Digital Systems

The proposed architecture for research methodology adopted in this article in order to obtain model reduction of digital systems using EFOS is detailed in the following section.

Let G be the original higher order ($m' \; n'$) transfer functionof the system and R is the resulted reduced system of order $m * n$ ($m < p$ & $n < q$). Here,the coefficients $a_0, ...a_m$ and $b_0, ...b_m$ are to be estimated such that the response of the original systemand the reduced system should be same. This can be done in two ways:

i) Using Δ method (Approximate Generalised Time Match- ing(AGTM))
ii) Send all these coefficients (referred as SC method, in this research) to any of the evolution algorithms such that the mean square error(y) of unit step response for original system and reduced system is minimum i.e. minimizing the function.

$$G(z) = \frac{bo+b_1z+\cdots\cdots+b_pz^p}{ao+a_1z_i\cdots\cdots+a_qz^q} \quad (1)$$

$$R(z) = \frac{bo+b_1z+\cdots\cdots+b_mz^m}{ao+a_1z_i\cdots\cdots+a_nz^n} \quad (2)$$

$$\mathrm{Y} = \frac{1}{n}\sum_{i=0}^{n}(g[i]-r[i])^2 \quad (3)$$

where g and r are step responses of original and reduced systems respectively, n is the order of reduced system, and y is the mean square error (MSE). Here instead of finding δ we find all the coefficients of the reduced model R by sending these coefficients to any of the evolutionary algorithm with only condition that mean square error of the step response for the original and the reduced system is minimized.

4. RESULTS AND DISCUSSION

The proposed technique for model reduction Ensemble Framework for Optimized System (EFOS) has been implemented and mathematically verified. The results obtained and the discussion thereof, is covered in this section of the research article.

4.1. Results Of Model Reduction from Using Δ Method (AGTM)

Here reduced model *(R)* is written as follows:

$$R(z) = gain * \frac{1+b_1\,z+\cdots\cdots+b_mz^m}{1+a_1z+\cdots\cdots+a_nz^n} \quad (4)$$

at $z = p_i = 1 + \Delta * I$

G(z) = R(z) where i= 1,2,3…m+n-1 (5)

Therefore,

$$G(p_i) = gain * \frac{1+b_1\,p_i+\cdots\cdots+b_mpi^m}{1+a_1(p_i)+\cdots\cdots+a_n(p_i)^n} \quad (6)$$

Let $G(p_i) = t_i$

$$t_i\left(1+\sum_{j=1}^{m}a_j\right) = gain * \left(1+\sum_{j=1}^{n}b_j\right) \quad (7)$$

Once these matrices are evaluated,

$\boldsymbol{X} = \boldsymbol{A}^{-1} * \boldsymbol{B}$ Where $\boldsymbol{X}$ is reduced model unknown coefficients. (8)

This **Δ** is estimated through the following algorithms while taking transfer function G(z) of the system as: (A standard example is taken from [11])

$$G(z) = (0.98-5.09z + 10.02z^2 - 8.88z^3 + 3z^4) / (-0.21 + 1.48z - 4.03z^2 + 5.47z^3 - 3.7z^4 \quad (9)$$

(1) Genetic Algorithm: System is reduced to 2nd order (R)using GA (to find *δ*).The parameters are as follows :

(a) Probability of Crossover = 0.8, u = 20
(b) Probability of Mutation = 0.2 , n = 20
(c) Number of chromosomes = 50
(d) Number of Generations = 15

The min Mean Square Error is obtained at Δ = - 0.0066 Fig. 6 and Fig. 7 represents Del values vs mean square errorconvergence as generation proceeds for GA.

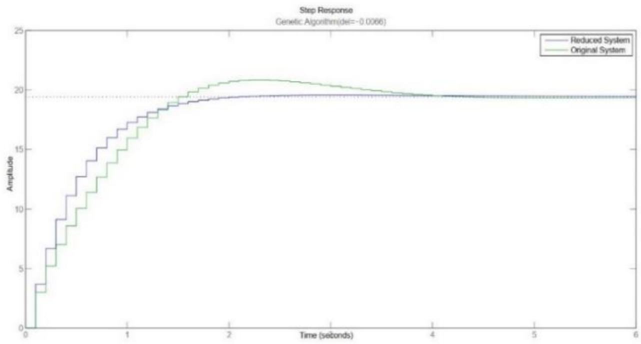


Figure 5. Step Responses for GA: Original (Green) and Reduced (Blue)

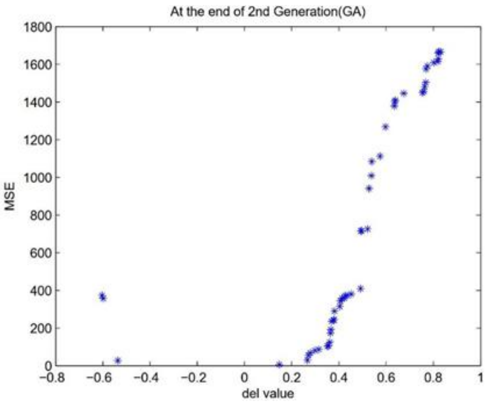


Figure 6. Del values after 2nd Generation

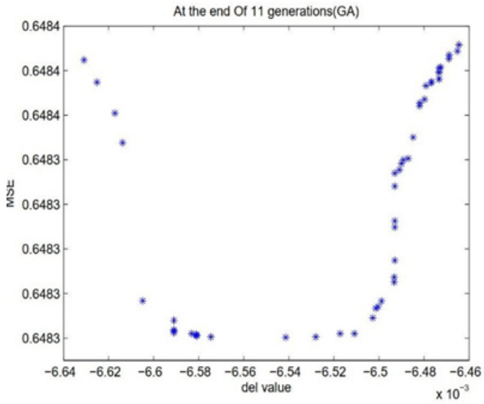


Figure 7. Del values after 11th Generation

2) Particle Swarm Optimization: If transfer function of original system is G(z), The system is reduced to 2nd order

(R) using PSO algorithm (to find δ).The parameters are as follows:

(a) Number of chromosomes = 50.

(b) Number of Generations = 35

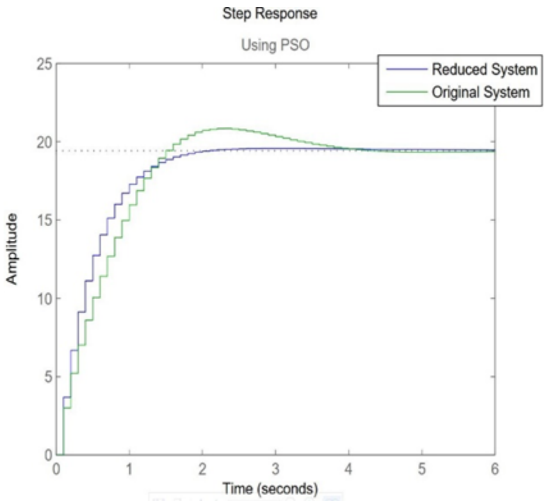
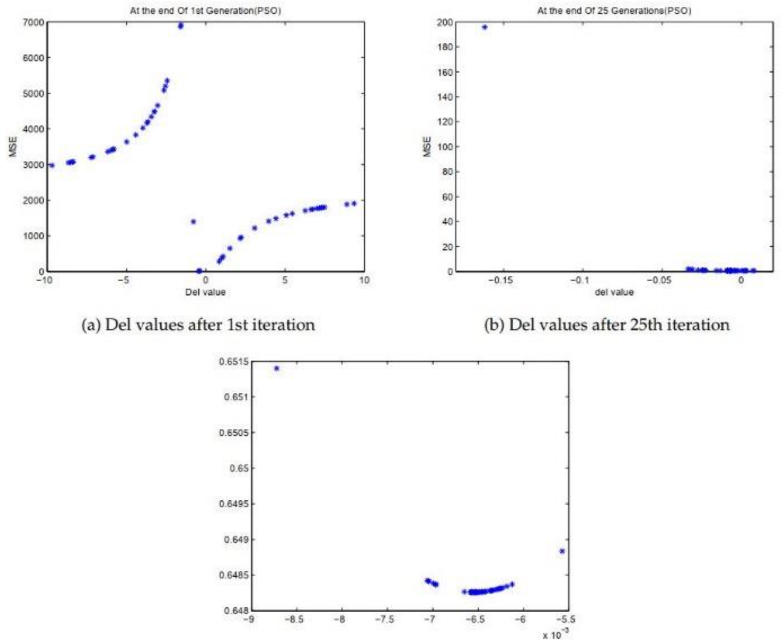


Figure. 8. Step Responses for PSO algorithm: Original (Green) and Reduced (Blue)

Figure 8 and 9 depicts results for PSO containing step response and del values after 1st Gen, 25th iteration and 33rd iteration respectively.



(a) Del values after 1st iteration (b) Del values after 25th iteration

(c) Del values after 33rd iteration

Figure 9. Del Value convergence as iteration proceeds for PSO

3) Differential Evolution: If transfer function of original system is G(z), The system is reduced to 2nd order (R) using DE algo (to find δ).The parameters are as follows:

(a) Number of chromosomes = 50.
(b) Number of Generations = 35
(c) Crossover Ratio = 0.9.
(d) Scaling Factor = 0.4

The min Mean Square Error is obtained at Δ = - 0.0066.

Figure 10 and 11 depicts results for DE algorithmcontaining step response and del values after 1st Gen, 25th iteration and 33rd iteration respectively.

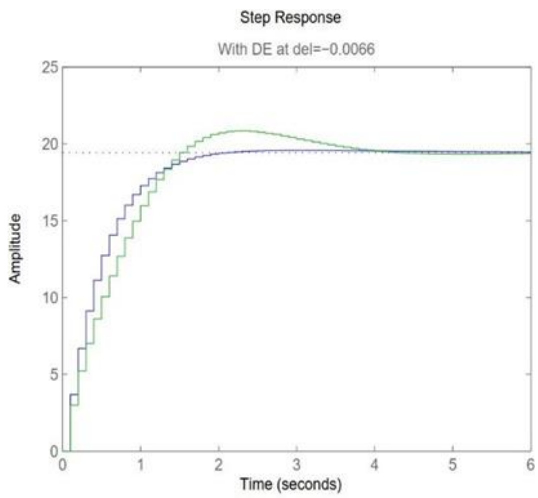
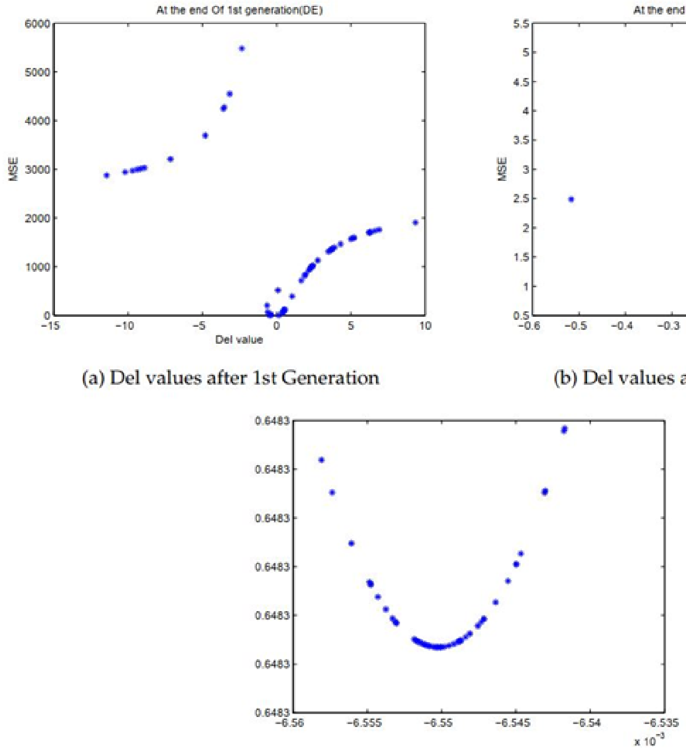


Figure 10. Step Responses for DE algorithm: Original (Green) and Reduced (Blue)

4) Luus Jaakola Optimization*:* If transfer function of orig-inal system is G(z), The system is reduced to 2nd order using Luus Jakola Optimisation algorithm (to find δ).Theparameters are as follows:

(a) Number of chromosomes = 1
(b) Number of Generations = 170

Figure 12 is the step response for LJO algorithm. The minMean Square Error is obtained at $\Delta = -0.0056$



(a) Del values after 1st Generation

(b) Del values after 25th Generation

(c) Del values after 33rd Generation

Figure 11. Del values convergence as generation proceeds for DE algorithm

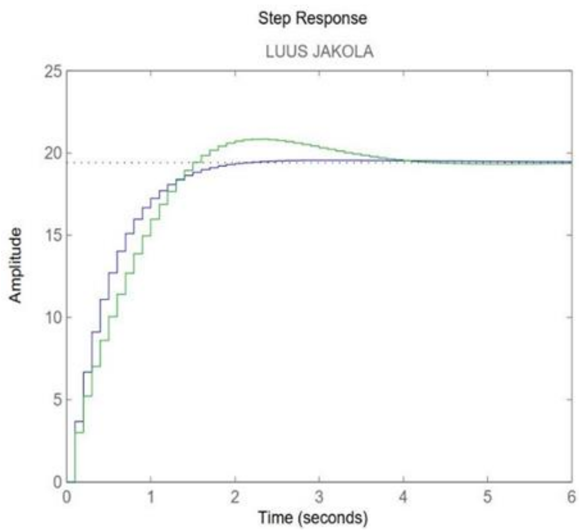


Figure 12. Step Responses for LJO algorithm

5) Ensemble Framework for Optimized System (EFOS) : Proposed Model comprising Luus Jaakola Optimization + Genetic Algorithm Method

In certain cases, it is seen that Luus Jaakola Optimization converges to a local minima than the requisite global minima due to vague initialisation of population and considerable error is seen. To avoid this, we use GA upto 3 generations and the resulted population is sent as the initialisation vector to Luus Jaakola algorithm. The learnings obtained from Genetic Algorithm is sent to Luus Jaakola algorithm to further optimise performance and speed of the result for the reduced model. This method is newly devised method of this research endeavour and be further called 'Transfer Learning, EFOS method' in this research article. Figure 13 represents Step Response for Transfer Learning Algorithm. The results of Transfer Learning method are discussed in Section 5.

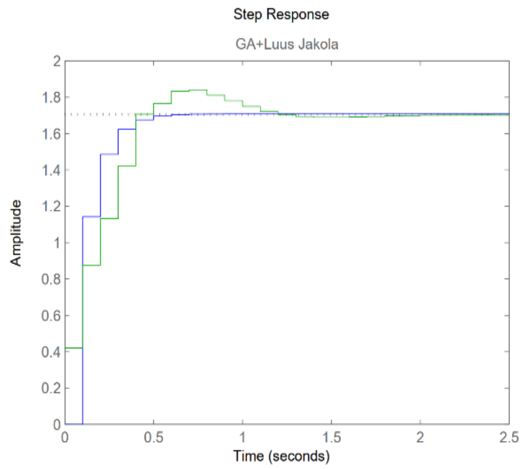


Figure 13. Step Responses for Original (Green) and Reduced (Blue) for EFOS method Luus Jaakola + GA (For 3 Generations)

4.2. Results Of Model Reduction From SC Method

The results mentioned in this section is achieved by takingthe transfer function *G(z)* same as Eqn.11.

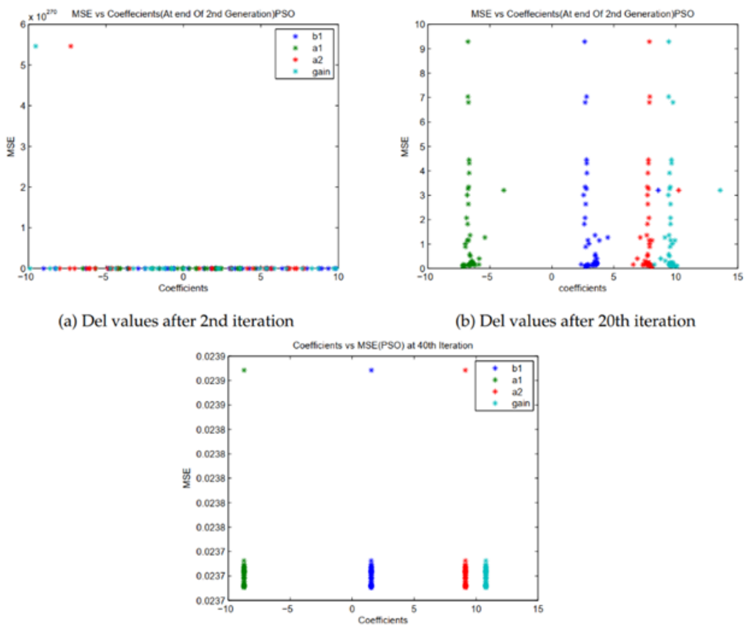
A. Particle Swarm Optimisation

The min Mean Square Error is obtained for coefficients:

$$b_1 = 1.5348, a_1 = -8.7338, a_2 = 9.1416, gain = 10.7885$$

These variables are in accordance to Eq. 4. Parameters takeninto consideration were:

1) Number of chromosomes = 50
2) Number of Generations = 40
3) Number Of Variables = 4 Figure 14 represents coefficient value convergence after various number of iters for PSO algorithm



(a) Del values after 2nd iteration (b) Del values after 20th iteration

Figure 14. Coefficient values convergence as iteration proceedsfor PSO algo

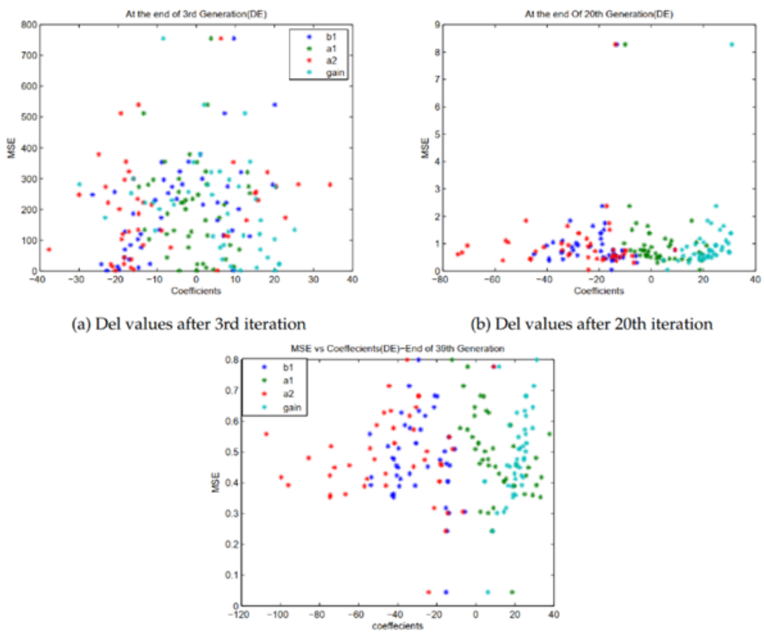
B. Differential Evolution

The min Mean Square Error is obtained for coefficients:

b1 = -15.2670, a1 = -18.6376, a2 = -24.1345, gain = 6.1566 Parameters taken into consideration were:

1) Number of chromosomes = 50
2) Number of Generations = 35
3) Crossover Ratio=0.9.
4) Scaling Factor=0.4

Figure 15 represents del value convergence after variousnumber of iters for DE algorithm



(a) Del values after 3rd iteration (b) Del values after 20th iteration

Figure 15. Coefficient values convergence as iteration proceedsfor DE algorithm

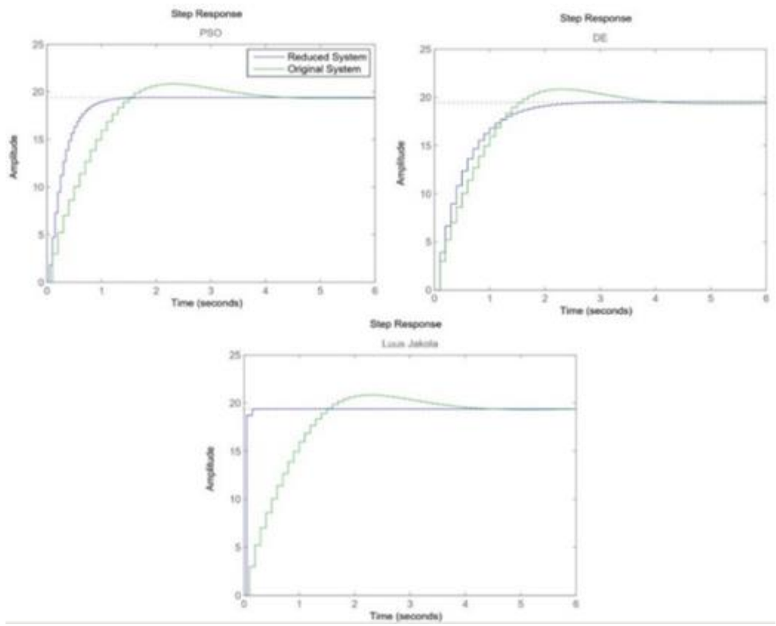


Figure 16. Step Responses for various algorithm: (a) PSO Algorithm, (b) DE Algorithm, (c) LJO algorithm

C. Luus Jaakola Optimisation

The min Mean Square Error is obtained for coefficients:

b_1 = - 15.2670, a_1 = 18.6376, a_2 = - 24.1345, *gain* = 6.1566 Parameters taken into consideration were:

1) Number of chromosomes = 1
2) Number of Generations = 170

Figure 16 depicts step response of various algorithms used for second method illustrated in this research, by Complete Sending of coefficients method.

4.3. Digital Control Design

If $G_p(s)$ is the plant for which digital controller is intended, and ZOH represents zero order hold at sampling time Ts, then

$$G_p(s) = (ZOH) * G_p(s) \quad (10)$$

determines the digital equivalent of the plant.

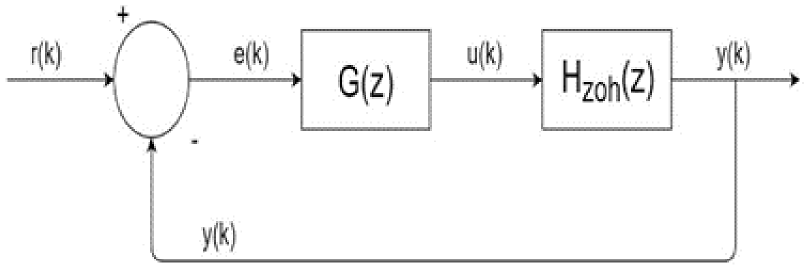


Figure 17. Digital Controller basic flow diagram

Let C(z) be the controller to the plant, M(z) be the transfer function of a system obtained from the required specifications of the system (Peak Overshoot, Settling Time, Peak Time etc.). Then,

$$M(z) = \frac{G_{ph}(z)*C(z)}{1+C(z)*G_{ph}(z)} \quad (11)$$

$$C(z) = \frac{M(z)}{G_{ph}(z)*(1-M(z))} \quad (12)$$

Now that C(z) is formed,controller is designed. This method of designing the controller is called Exact Model matching (Truxal's Method) has the following disadvantages:

1) Depending on the complexity,number of poles and zeros of $G_{ph}(z)$ are M(z), C(z).
2) Higher order (sum of orders of $G_{ph}(z)$ and M(z)), C(z) may be unstable because of the the formation of new poles in the RHP.
3) Controller may not be realisable because there is a possibility of formation of more number of poles than zeros.

The alternative is to design the controller with the approxi- mate model matching using Approximate Generalised Time Matching (AGTM) and Approximate Generalised Markov Process (AGMP). The steps are as follows:

Step I: Find equivalent open loop model $M_q(z)$ in accor- dance with the above diagram, where M(z) is the desired model. Therefore, Figure 18 represents these equations in a control system

$$M(z) = \frac{M_n(z)}{(1+M_n(z))} \quad M(z) = \frac{M(z)}{(1-M(z))} \quad (13)$$

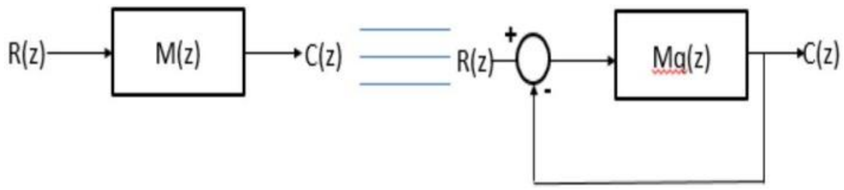


Figure 18. Digital Controller where $H_{zoh}(z) = G_{ph}(z)$

Step II: Let, $$X(z) = \frac{M(z)}{(G_{ph}(z)}$$ (14)

Then, equation which follows is below:
C(z) = X(z)
Choose an nth order controller C(z), where $n \le m$ such that

$$C(z) = \frac{co + c_1\, z + \cdots\cdots + c_m z^m}{do + dz + \cdots\cdots + d_n z^n} \quad (15)$$

Step III: Now we will do AGTM to find the controller coefficients. At $z = z_k = 1 + \Delta * k$, where k = 0,1,2,3,...2n

$$C\ (z_k) = X\ (z_k) \quad (16)$$

With this the controller is designed. The exact δ value is obtained through the optimisation algorithms like Genetic Algorithm, PSO (Particle Swarm Optimization) , DE (Differ- ential Evolution) algorithms such that the mean square error(Y) between the step responses of the desired model and the reduced is minimized.

4.4. Results Of Digital Control Design

The plant for which a digital controller is designed was $$G(s) = \frac{1+s}{(1+5s)(1+3.5s)(1+1.5s)} \quad (17)$$

The design specifications are:
1) Peak Overshoot M_p < 10%
2) Peak Time t_p < 6 sec
3) Settling Time t_S < 10
4) Steady state error nearly 0
5) 3dB bandwidth = 0.78 rad/sec

The desired model was estimated to be:
$C(z) = \frac{Az+B}{z^2+Cz+D}$ where A = 0.103, B = 0.028, C = -1.424, D = 0.555

At Ts = 0.5 sec, The plant model (with zero order hold) was estimated to be:

$$C(z) = \frac{bo + b_1\, z + b_2 z^2}{ao + az + a_2 z^2 + z3} \quad (18)$$

where b_2 = 4.6228* 10^{-3}, b_1 = 1.69947*10^{-3}, b_0 = - 2.73126*10^{-3}, a_2 = -2.488247, a_1 = 2.0538732, a_0 = - 5.62035*10^{-1}
A PID Controller Controller C(z) is designed to this system to meet the required specifications

$$C(z) = \frac{co + c_1\, z + c_2 z^2}{z(z-1)} \quad (19)$$

Here the coefficients c_0, c_1, c_2 are estimated with AGTM matching and the δ value was obtained using Particle Swarm Optimization and the combination of Luus Jaakola and GA.

The min Mean Square Error is obtained at Δ = 0.2544.
And the desired controller is

$$C(z) = \frac{15.65 - 36.17\, z + 20.772 z^2}{z(z-1)} \quad (20)$$

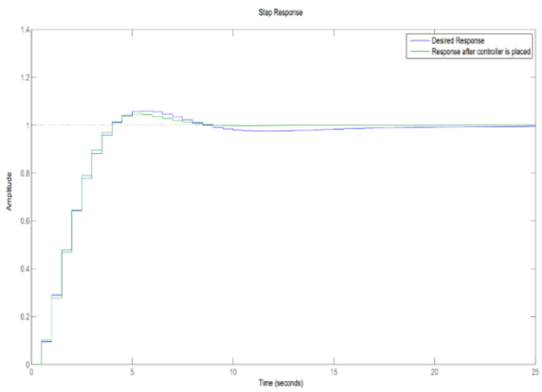


Figure 19. Step Responses of PID controller for Model

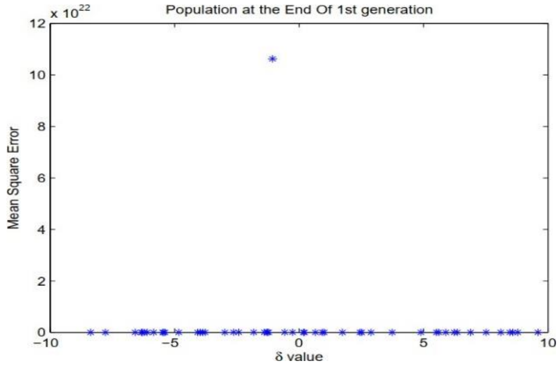


Figure 20. Delta values after second generation (in Blue) and to the plant with controller (in Green)

The delta values after multiple iterations [Second iteration, Twentieth iteration and Thirtieth iteration] have been depicted in Fig 20 to Fig 21.

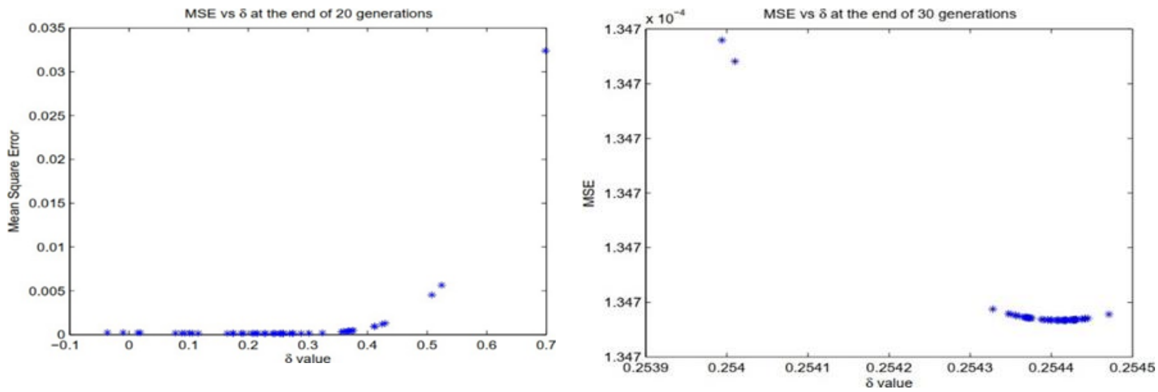


Figure 21. Delta values after twentieth and thirtieth generation

5. DISCUSSION

This section presents insights and comparison between various described models on results as found in Section 4.

5.1. Model Reduction

1) Using δ method

If

$$G(z) = \frac{0.98-5.09z+10.02z^2-8.88z^3+3z^4}{-0.21+1.48z-4.03z^2+5.47z^3-3.7z^4} \quad (21)$$

The most appropriate reduced order model (ROM) is

$$R(z) = \frac{4.515+4.833z}{-1-0.2443z+1.717z^2} \quad (22)$$

Table I illustrates the comparison of performance of various algorithms of Model Reduction for this model

Example II:
If G(z) = A/B

Table 1. Comparison of performance of various algorithms of Model Reduction for example in Equation 11

Algorithm	Time Taken (sec)	δ	MSE	Generations
GA	542.81	-0.0066	0.6483	15
PSO	507.93	-0.0066	0.6483	35
DE	728.26	-0.0066	0.6483	35
Luus Jaakola	51.33	-0.0066	0.6483	170

where,

$A = 1 + 7.3z^1 - 43.6z^2 + 86z^3 + 25.3z^4 - 35z^5 + 186z^6 + 280.3z^7$
$B = 1 - 7.3z + 43.6z^2 + 86z^3 - 25.3z^4 - 186z^5 - 280.3z^6 + 666.6z^7$

The most appropriate reduced order model (ROM) is

$$R(z) = \frac{49\,z-1}{43z^2-13.96z-1} \tag{23}$$

Table II illustrates the comparison of performance of various algorithms of Model Reduction for this model.

Table 2. Comparison of performance of various algorithms of Model Reduction for example in Equation 11

Algorithm	Time Taken (sec)	δ	MSE	Generations
GA	555.42	0.0047	0.0703	15
PSO	639.05	0.0047	0.0703	35
DE	819	0.0047	0.0694	35
Luus Jaakola	49.71	0.0047	0.0706	170

2) SC method:
Table III illustrates the comparison of performance of various algorithms of Model Reduction for SC Model (Complete sending of coefficients)

Table 3. Comparison of performance of various algorithms of Model Reduction for example in Equation 11

Algorithm	PSO	DE	Luus Jaakola
Time Taken (sec)	557.54	1061	52.82
MSE	0.00237	0.0446	0.4950
Gain	10.7	6.1	24

3) Proposed method: Ensemble Framework for Optimized System (EFOS) for Model Reduction
Transfer Learning method gave much smaller MSE and took far less time compared to GA, thus taking best of both algorithms, GA and LJ. The performance indices obtained in TL approach are as follows:

a. δ value obtained is 0.0702
b. MSE = 0.0047
c. Elapsed time is 158.35 seconds (Far Less than with Only GA which was 560 seconds).

The comparative analysis of the proposed Ensemble Framework for Optimized System (EFOS) for Model Reduction in terms of MSE and time taken is presented in Figure 22. Further, the comparative analysis of MSE×Time is depicted in Figure 23.

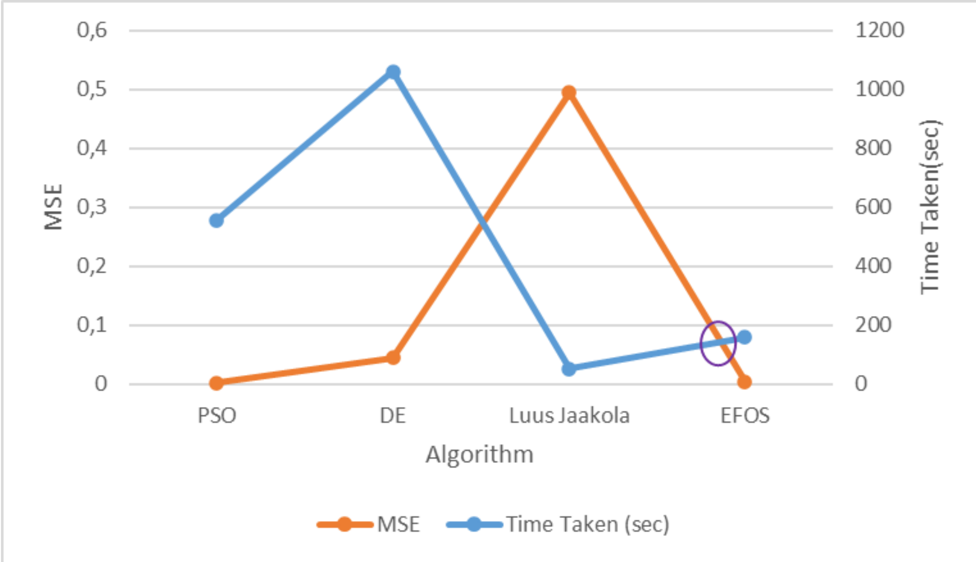


Figure 22. Comparative analysis of Ensemble Framework for Optimized System (EFOS)

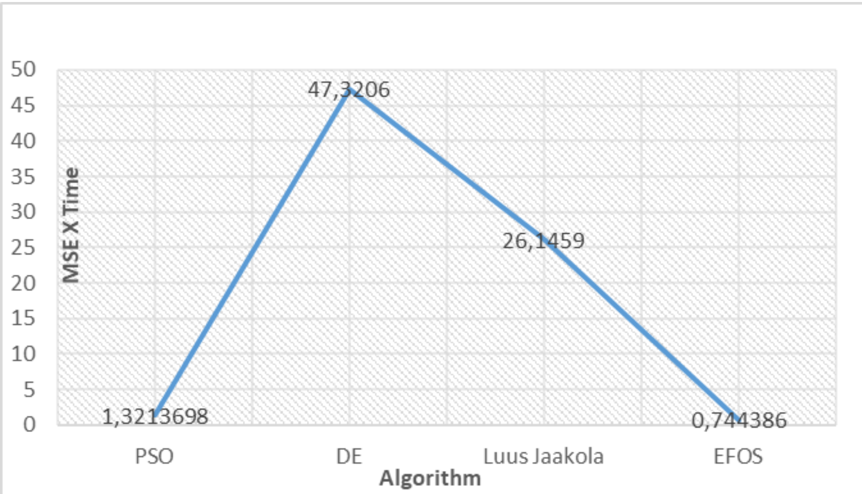


Figure 23. Comparative analysis of MSE×Time

The comparative analysis reveals that out of all the meta heuristic approaches the proposed EFOS approach provides the exceptionally better performance both in terms of MSE and time taken. The comparison reveals the reliability of the proposed transfer learning-based approach, EFOS. While combining the advantages of Luus Jaakola and Genetic algorithms for EFOS it is depicted that their individual counterparts on diverse performance parameters like speed of convergence and optimal convergence to global minima.

5.2. Digital Controller Design

The comparison of various algorithms for design of Digital Controller have been mentioned in Table IV. It can be therefore, concluded proposed Ensemble Framework for Optimized System (EFOS) exhibit better results for the design of Digital controller in terms of performance metrics.

Table 4. Comparison of performance of various algorithms for Digital Controller Design

Algorithm	Time Taken (sec)	δ	MSE	Generations
PSO	199.978	0.2544	0.00134	30
EFOS(Proposed Method)	29.7189	0.2544	0.00142	170

6. CONCLUSION

This research article employs various heuristic algorithms for SISO System model reduction and also for Digital Con- troller Design and their performance measures were evaluated using two methds, δ method (i.e. AGTM Matching) and Complete Sending of variable method. It was concluded that Luus Jaakola Optimization algorithm is way faster incomparison with the other algorithms. The only problem with Luus Jaakola is that, its solution converges to a local minima. To avoid this, the initial population is generated with theoutput of Genetic Algorithm after 3 generations. This method converged to a global minima. It was seen that the elapsed time to run this algorithm nearly reduced to five times thatof using, only GA. The proposed algorithm Ensemble Framework for Optimized System (EFOS) performs much more efficiently in terms of speed Mean square error, Delta value. The percentage improvement in the time taken for design of digital controller is 85.3%, with no change in delta value

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