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FIR Filter Design using Raised Semi-ellipse Window Function

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ABSTRACT

In this paper, a new two-parameter window function - Raised semi-ellipse (RSE) is proposed. The window is obtained from a fixed elliptical window known as Semi-ellipse window by raising the radius of the minor axis by the parameter (β), and applied for the design of finite impulse response (FIR) digital filters. The spectral parameters of the proposed window are determined first and compared with the Kaiser window – a 2-parameter adjustable window. Subsequently, in its application in filter design with an established design algorithm, the newly proposed adjustable window is compared to the Semi-ellipse window to examine its improvement and also the Kaiser window to compare its performance with a commonly used adjustable window. The filter simulation results show that the filters designed with the proposed window can provide more reduced ripples than the Kaiser window for prescribed spectral characteristics.

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1. INTRODUCTION

Digital filters are used in signal processing for reducing or enhancing certain parts of a signal. Hence, a digital filter is a system that receives discrete-time signal input and produces output in the same signal format [1]. For astounding performance of digital filters, digital signal processing (DSP) has become prevalent in the field of digital electronics. Digital filters can be classified based on time of their impulse reponse as either finite impulse response (FIR) or infinite impulse response (IIR) filters. However, FIR digital filters have some application advantages, which include possible exact linear-phase characteristic and guaranteed filter stability [2, 3]. Its disadvantage is basically computational complexity. This drawback of FIR digital filters can be minimized by the application of fast numerical algorithm like FFT in the implementation [2]. Finite impulse response filters are designed using different methods which include window design method, frequency sampling method and numerical method all which are based on ideal filter approximation [2, 4]. Window technique is the most conventional and simple method used for designing FIR filter. A window can be referred to as an array of filter coefficients that satify prescribed filter specifications [5, 6]. Hence, the window method is used to remedy the unwanted Gibbs's oscillations [4-6] in the designed FIR filter.

Moreover, categorization of windows as fixed or adjustable is determined by the number of independent parameters present in the function [7]. Fixed windows have only the window length as the independent parameter. Rectangular, Bartlett window, Von Hann, Hamming, Blackman windows are some fixed windows in literature [4-6, 8] and Semi-ellipse window [9] which is developed lately. Unlike the fixed window, an adjustable window uses more than one independent parameter to control the window characteristics besides the window length. Some adjustable window functions like the Dolph Chebychev window [10], Kaiser Window [11], and Ultraspherical window [12] are proposed earlier. Furthermore, combination of window functions facilitates the production of hybrid window function that has enhanced characteristics in the spectrum for various filter applications. Among the hybrid windows are Bartlett-Hann [6, 8] and Blackman-Harris [13]. A comprehensive list of window functions is available in [8]. Lately, a combination of existing windows that improved spectral characteristics for various filter application were proposed by the authors in [14 - 16]. However, selection of window function is depedent on that which satisfies the presented filter specification.

A new adjustable window function termed the Raised semi-ellipse (RSE) window is presented. It is developed from the Semi-ellipse window. The filtering performance is compared with the Kaiser window and the Semi-ellipse window to examine the improvement of RSE window. In section 2, the methodology for developing the proposed window function is introduced after brief explanations on the window theory. Subsequently, the efficient formulations for computing the coefficients of the RSE window and its spectral parameters are presented. The new window function is used in the design of FIR digital filters with the Kaiser window and the Semi-ellipse window thus demonstrating the application of design algorithm. In Sections 3 and 4 the discussion of results and conclusion are presented respectively.

2. METHODOLOGY FOR DEVELOPING THE NEW WINDOW FUNCTION

2.1. The Window Spectral Characteristics

The design of FIR filters from window functions (or simply windows) starts by evaluating the impulse response, h(n) of the filter using a given desired frequency response, $H(e^{j\omega})$. Relationship between impulse response and magnitude response is shown in Eq. (1) and Eq. (2) as illustrated in [2].

$$H(e^{j\omega}) = \sum_{n=\infty}^{\infty} h(n)e^{-j\omega n} \quad (1)$$

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega n}d\omega \quad (2)$$

In the time domain, FIR windowing as in Eq. (3) simply means multiplying the infinite-duration impulse response, h(n) with a window function, w(n) of length, N to generate a finite-length impulse response filter, $h'(n)$ yielding filter order of N-1. The coefficients of the window are determined using the window function, w(n).

$$h'(n) = h(n) * w(n); \quad 0 \le n \le N-1 \quad (3)$$

2.2. The Raised Semi-Ellipse Window Function

The proposed Raised semi-ellipse window is developed from the fixed semi-ellipse window. The Semi-ellipse window function is derived from the equation of an ellipse using two formats which produced equivalent results as illustrated in [9]. These equations are illustrated in [17] and are given by Eq. (4) in explicit form and Eq. (5) in parametric form as depicted in Figure 1.

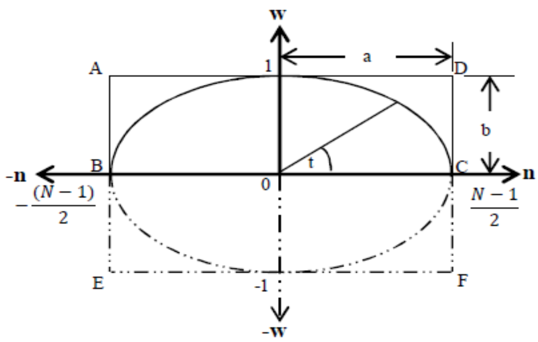


Figure 1. An ellipse window showing the radius of the major axisand the radius of the minor axis[9]

$$\frac{(n+h)^2}{a^2} + \frac{(w+k)^2}{b^2} = 1 \quad (4)$$

$$n = a\cos t \quad (5a)$$

$$w = b\sin t \quad (5b)$$

The causal form of the Semi-ellipse window function derived from the explicit equation as in Eq. (4) and the parametric equation as in Eq. (5) is as given in Eq. (6) and Eq. (7), respectively, and the details of these computations are as in [9].

$$w_{se}(n) = 2\frac{\sqrt{((N-1)n-n^2)}}{N-1}; \quad \begin{cases} 0 \le n \le N-1 \\ 0 \quad \text{otherwise} \end{cases} \tag{6}$$

$$w_{se}(n) = \sin\left(\arccos\left(\frac{2n}{N-1} - 1\right)\right); \quad \begin{cases} 0 \le n \le N-1 \\ 0 \quad \text{otherwise} \end{cases} \tag{7}$$

Equations (6) and (7) are mathematically equivalent and can be used interchangeably.
However, the Raised semi-ellipse window function is an elevated Semi-ellipse window with a value of beta, β as shown in Figure 2. The β value ranges from 0 to 1.

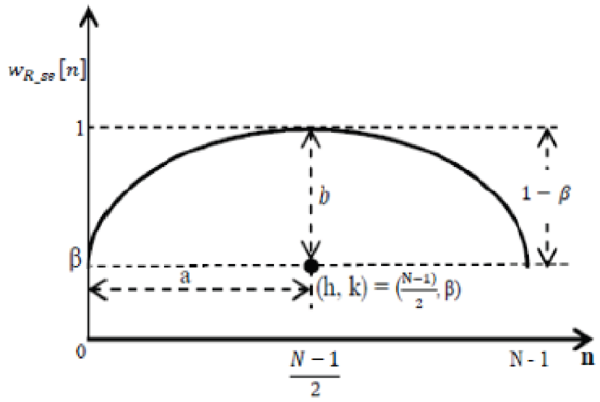


Figure 2. A semi-ellipse raised with the value of β along the minor axis

Refer to Figure 2, the Raised semi-ellipse window function derived from Eq. (4) is given by:

$$w_{R\_se}(n) = \beta + \frac{2(1-\beta)}{N-1}\sqrt{(N-1)n - n^2}; \quad \begin{cases} 0 \le n \le N-1 \\ 0 \quad \text{otherwise} \end{cases} \tag{8}$$

where $0 \le \beta \le 1.0$

Similarly, the Raised semi-ellipse window function derived from the parametric equation of an ellipse, refer to Eq. (5), is given by:

$$w_{R\_se}(n) = \beta + (1-\beta)\sin\left(\arccos\left(\frac{2n}{N-1} - 1\right)\right); \begin{cases} 0 \le n \le N-1 \\ 0 \quad \text{otherwise} \end{cases} \tag{9}$$

where $0 \le \beta \le 1.0$

Again, Eqs. (8) and (9) are mathematically equivalent and can be used interchangeably.

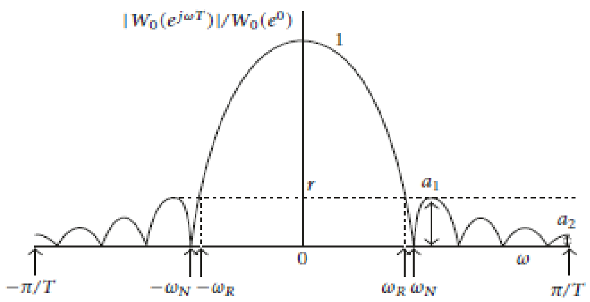


Figure 3. A typical window in the frequency domain [18]

In the frequency domain, a window is described by two parameters, main-lobe width and the ripple ratio. As depicted in Figure 3, the main-lobe width is equal to ‘$2W_R$’ and the ripple ratio, also referred to as peak amplitude of the side-lobe, is equal to ‘r’. Like the Kaiser window, the Raised semi-ellipse window has two parameters: the length, N and a shape parameter β. By varying N and β, the window length and shape can be adjusted to trade side-lobe amplitude for main-lobe width. Figure 4(a) shows the shape of RSE window of length N = 51 for β = 0, 0.35, 0.77 and 1.0. Notice from Eq. (10) that for case β = 0 it is the semi-ellipse window and becomes the rectangular window when β = 1. Figure 4(b) shows the corresponding frequency domain (Fourier transform) of the RSE windows in Figure 4(a).

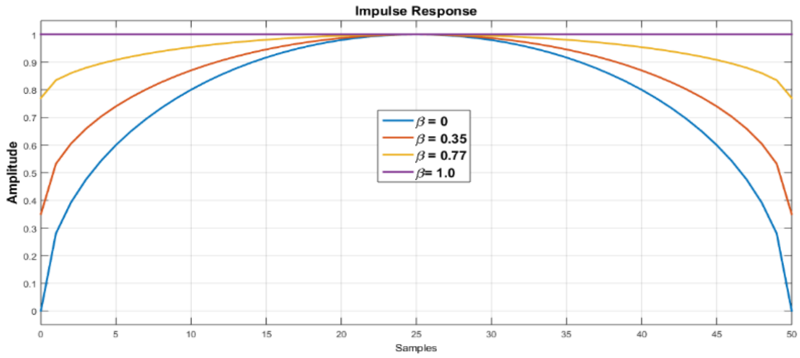


Figure 4(a). The impulse response of RSE window for different values of β for N = 51

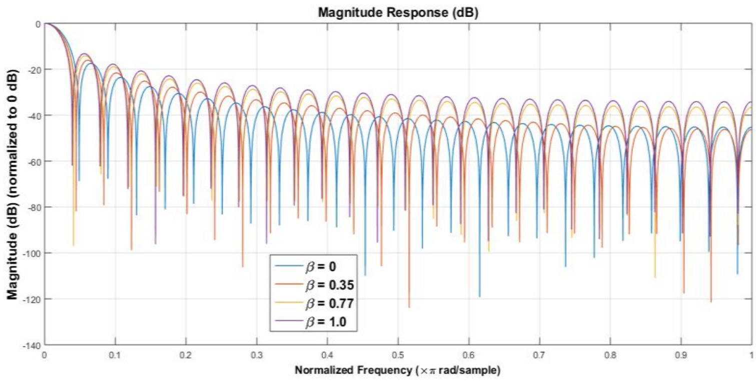


Figure 4(b). The magnitude response of RSE window for different values of β for N = 51

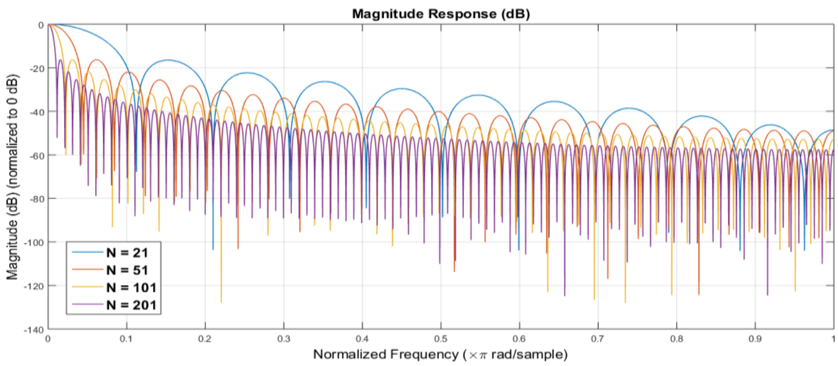
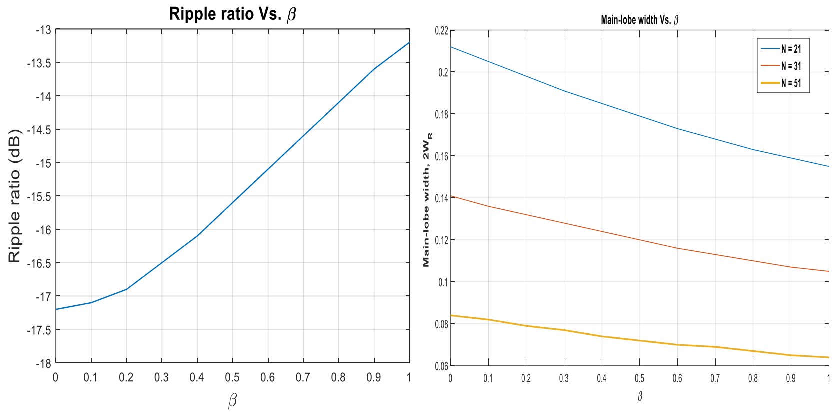


Figure 4(c). The magnitude response of RSE window for different values of N for β = 0.3

Figure 4(c) shows the frequency domain of RSE windows with β = 0.3 and N = 21, 51, 101 and 201. The plots in Figures 4(a) and (b) clearly show that if the window is raised more, the sidelobes of the Fourier transform become bigger, but the main-lobe becomes narrower while Figure 4(c) shows that as N increases with constant β results in decrease in width of the main-lobe without affecting side-lobe maximum amplitude.

Figure 5(a) and (b) shows the plot of the ripple ratio and main-lobe width of the RSE window as functions of β. Figure 5(a) shows that the ripple ratio increases rapidly as β is increased but it is practically independent of the window length. On the other hand, Figure 5(b) shows a decrease in the width of the main-lobe with increase in values of β.



(a) (b)

Figure 5. The characteristics of RSE window: (a) Ripple ratio in dB versus β for N = 21, (b) main-lobe width versus β for different window lengts.

Therefore, by a process of numerical experiments, we developed a pair of expressions which allows filter designers to predict in advance the values of N and β required to meet a given frequency-selective filter specification. We determined empirically that the value of β required for realizing specified stopband attenuation for the RSE window to be given by;

$$\beta = \begin{cases} 0; & A_s = 27 \\ 0.1667(27 - A_s); & 21 < A_s < 27 \\ 1; & A_s = 21 \end{cases} \quad (10)$$

Recall that the case β = 0 is the Semi-ellipse window for which $A_s = 27$ and case β = 1 is the rectangular window for which $A_s = 21$. Besides, we discovered that to obtain desired values of stopband attenuation, A_s and transition width, $\Delta\omega$, N must satisfy Eq (11)

$$N \approx \frac{2.8(1-0.357\beta)}{\Delta\omega} \quad (11)$$

Equation (12) predicts that N can be computed through choosing the lowest odd value of N that satisfies Eq (12)

$$N \geq \frac{2.8(1-0.357\beta)}{\Delta\omega} \quad (12)$$

The filter length of the Semi-ellipse window required for the specified transition width can be determined as expressed in Eq. (13) [9] and rounded up to the nearest odd integer.

$$N \approx \frac{2.8}{\Delta\omega} \quad (13)$$

2.3. Relationship of the Algorithm for Raised Semi-ellipse Window with Kaiser Window in Filter Design

The Kaiser window is an adjustable window function which uses the window length, N and parameter β to vary the windows characteristics as in Eq (14) [11].

$$w_{kai}(n) = \frac{I_0\left(\beta.\sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0(\beta)}; \begin{cases} 0 \leq n \leq N-1 \\ 0 \quad otherwise \end{cases} \quad (14)$$

where $\propto = \frac{N-1}{2}$ and I_0 is a modified zero order Bessel function of the first kind.

$$\text{and, } \beta = \begin{cases} 0.1102(A_s - 8.7); & A_s > 50 \\ 0.5842(A_s-21)^{0.4} + 0.07886(A_s - 21); & 21 < A_s < 50 \\ 0; & A_s < 21 \end{cases} \quad (15)$$

where A_s = minimum stopband attenuation

The filter length, N can be determined by selecting the lowest odd value of N that would satisfy the inequality:

$$N \geq \frac{\omega_{samp}D}{\Delta\omega} + 1 \qquad (16)$$

Parameter D is determined using the expression:

$$D = \begin{cases} 0.9222; \; A_s \leq 21 \\ \frac{A_s - 7.95}{14.36}; \; A_s > 21 \end{cases} \qquad (17)$$

Where ω_{samp} is the normalized sampling frequency and $\Delta\omega$ is the transition width.

The ideal and realizable frequency responses of the four standard filters are shown in Figure 6 [6] and can be calculated using their corresponding functions as presented in Table 1 [3, 6].

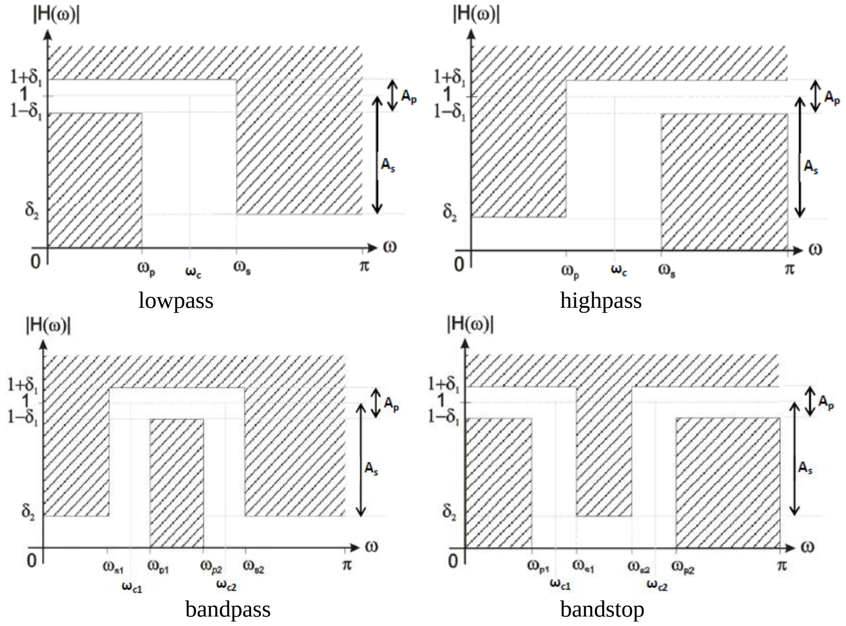


Figure 6. The frequency responses of the different filters

Table 1. The standard filters causal impulse response functions and cutoff frequency

Filter Type	Ideal Impulse Response function, $h_{id}(n)$	Cut-off frequeny, ω_c
Lowpass	$\begin{cases} \frac{\sin(\omega_c(n-M))}{\pi(n-M)}; \; n \neq M \quad (0 \leq n \leq 2M) \\ \frac{\omega_c}{\pi}; \quad n = M \end{cases}$	$\omega_c = \frac{\Delta\omega}{2}$ $\Delta\omega = \omega_s - \omega_p$
Highpass	$\begin{cases} -\frac{\sin(\omega_c(n-M))}{\pi(n-M)}; \; n \neq M \quad (0 \leq n \leq 2M) \\ 1 - \frac{\omega_c}{\pi}; \quad n = M \end{cases}$	$\omega_c = \frac{\Delta\omega}{2}$ $\Delta\omega = \omega_p - \omega_s$
Bandpass	$\begin{cases} \frac{\sin(\omega_{c_2}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c_1}(n-M))}{\pi(n-M)}; n \neq M \; (0 \leq n \leq 2M) \\ \frac{\omega_{c_2} - \omega_{c_1}}{\pi}; \quad n = M \end{cases}$	$\omega_{c_1} = \omega_{p_1} - \frac{\Delta\omega}{2}$ $\omega_{c_2} = \omega_{p_2} + \frac{\Delta\omega}{2}$ $\Delta\omega = \min[(\omega_{p_1} - \omega_{s_1}), (\omega_{s_2} - \omega_{p_2})]$
Bandstop	$\begin{cases} \frac{\sin(\omega_{c_1}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c_2}(n-M))}{\pi(n-M)}; n \neq M \; (0 \leq n \leq 2M) \\ 1 - \frac{\omega_{c_2} - \omega_{c_1}}{\pi}; \quad n = M \end{cases}$	$\omega_{c_1} = \omega_{p_1} + \frac{\Delta\omega}{2}$ $\omega_{c_2} = \omega_{p_2} - \frac{\Delta\omega}{2}$ $\Delta\omega = \min[(\omega_{s_1} - \omega_{p_1}), (\omega_{p_2} - \omega_{s_2})]$
	Note: $M = (N-1)/2$	Note: $\omega_c = \frac{2\pi f_c}{f_{samp}}$

2.4. Design Algorithm for Filter Design using Raised Semi-Ellipse Window and Kaiser Window

The algorithm of the design of FIR digital filter using RSE window starts by first suppling the specifications of the prescribed filter which is similar as in the case of the Semi-ellipse window as presented in [9]. The filter specifications may include the peak-to-peak passband ripple (A_p) as in (18), the minimum stopband attenuation (A_s) as in (19) and transition width (Δω) as in (20). The difference in the algorithm are the calculations for the parameter, β which is a determined using A_s as in (10), and the calculation of the filter length needed to design the filter which is determined using Δω and β as in (12).

$$A_p = 20\log\left(\frac{1+\delta_p}{1-\delta_p}\right) \tag{18}$$

$$A_s = -20\log\delta_s = -20\log\delta \tag{19}$$
where $\delta = min\{\delta_s, \delta_p\}$

Hence, $A_s = -20\log\delta$ (20)

Hence, the algorithm for the lowpass filter design using RSE window and that of the Kaiser window follows:

i. Supply filter specifications ω_s, ω_p, A_s and A_p.
ii. Calculate the transition width, Δω of the lowpass filter.
iii. Calculate the normalised cutoff frequency, ω_c which can be determined from the absolute cutoff frequency, f_c and sampling rate, f_{samp} as in Table 1.
iv. Compute the parameter, β according to the A_s of the prescribed filter specification for using Eq. (10) for RSE window and Eq. (15) for Kaiser window.
v. Calculate the filter length, N using Eq. (12) for RSE window and Eq. (16) for Kaiser window.
vi. Calculate the window function coefficients according to β and N obtained in Step iv and Step v, respectively using Eq. (8) or Eq. (9) for RSE window and Eq. (14) for Kaiser window.
vii. Calculate the ideal lowpass filter impulse response coefficients as in Table 1 in accordance with results in Step v.
viii. Calculate the causal finite-duration impulse filter as in Eq. (3) using results of Steps vi and vii.

For highpass, bandstop and bandpass filter design the lowpass design algorithm can be modified as depicted in Table 1.

Subsequently, Examples I and II are used to examine the performance of the Raised semi-ellipse window in FIR digital filters design for a lowpass filter and bandpass filter, respectively using the RSE design algorithm prescribed earlier.

Example I:

Design a lowpass filter that would satisfy the following specifications using Raised semi-ellipse, Kaiser and Semi-ellipse windows.

- Maximum passband ripple in frequency range 0 to 0.50π rad/sample: 1.1 dB
- Minimum stopband attenuation in frequency range 0.58π to 1.0π rad/sample: 21 dB
- Sampling frequency: 2π rad/sample

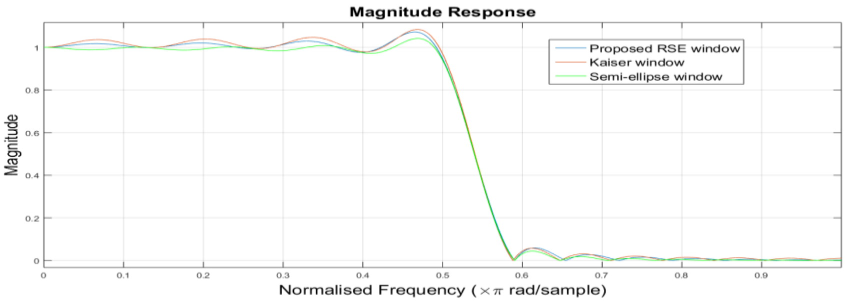
Solution:

i. Using Eq. (18) and Eq. (19), $\delta_p = 0.0632$ and $\delta_s = 0.0891$, respectively and using Eq. (20), $A_s = 23.986\ dB$.
ii. Given, $\omega_p = 0.50\pi\ and\ \omega_s = 0.58\pi\ (rad/sample)$; Δω = 0.08π (rad/sample)
iii. Calculate ω_c of the ideal lowpass filter as $\omega_c = 0.54\pi\ (rad/sample)$.
iv. Using Eq. (10) for RSE window and Eq. (15) for Kaiser window, computing the parameter, β according to the A_s of the prescribed filter specification, β for RSE = 0.502 and β for Kaiser = 1.140.
v. Using Eqs. (12), (16) and (13) the filter length, N required for the specified transition width are 29, 29 and 35 for RSE, Kaiser and Semi-ellipse windows, respectively.
vi. Calculate coefficients of the window function according to β and N obtained in Step iv and Step v, respectively using Eq. (8) or Eq. (9) for RSE window, Eq. (14) for Kaiser window, and Eq. (6) or Eq. (7) for Semi-ellipse window.

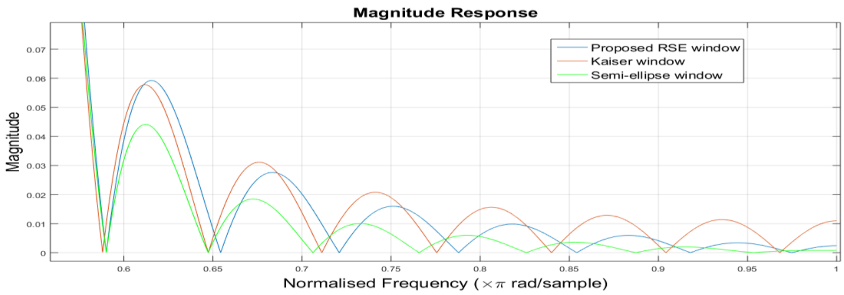
vii. Calculate coefficients of the ideal lowpass filter impulse response as in Table 1 using the results in Step v.

viii. Calculate the causal finite-duration impulse response using values obtained in Step vi and Step vii.

The plot of frequency responses of the filter using the windows for the parameters obtained were made using Matlab. The minimum stopband attenuation obtained for RSE, Kaiser, and Semi-ellipse were -24.55, -24.77 and -27.10 dB; the transistion widths achieved were 0.0783π, 0.0729π and 0.0849π rad/sample, respectively. The designed lowpass filter's magnitude responses are shown in Figure 7.



(a)



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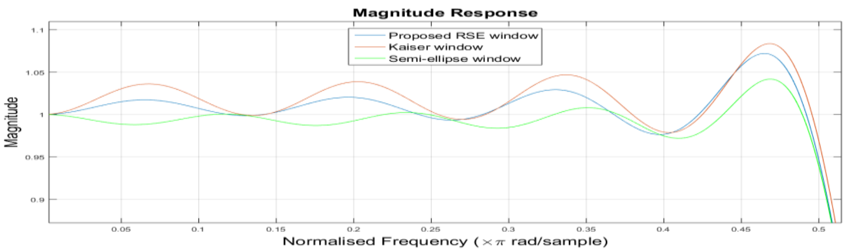


(c)

Figure 7. Magnitude to Frequency plots of Example I (a) lowpass filters, an enlarged view of: (b) stopband attenuation and (c) passband ripples

Example II:

Design a bandpass filter using RSE, Kaiser and Semi-ellipse windows that satisfies given specifications:

- Minimum attenuation for $0 \leq f \leq 100\ Hz$: $22\ dB$
- Maximum passband ripple for $150 \leq f \leq 300\ Hz$: $1.5\ dB$
- Minimum attenuation for $325 \leq f \leq 500\ Hz$: $22\ dB$
- Sampling frequency: $1000\ Hz$

Solution:

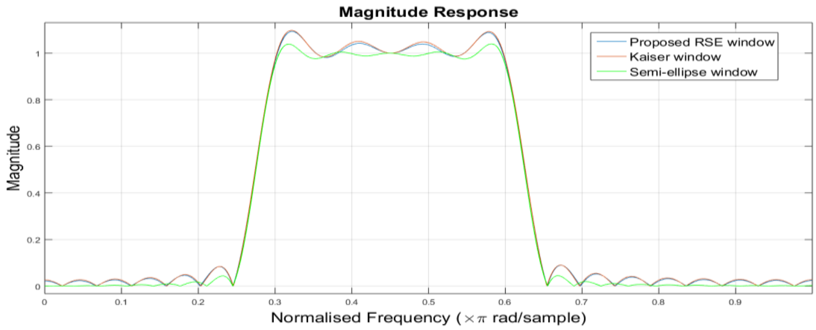
i. As ealier presented, $\delta_p = 0.086133$ and $\delta_s = 0.079432$ and $A_s = 22.0\ dB$

ii. From Table 1, $\omega_{s_1} = 0.2\pi$, $\omega_{p_1} = 0.3\pi$, $\omega_{p_2} = 0.6\pi$, and $\omega_{s_2} = 0.65\pi\ rad/sample$

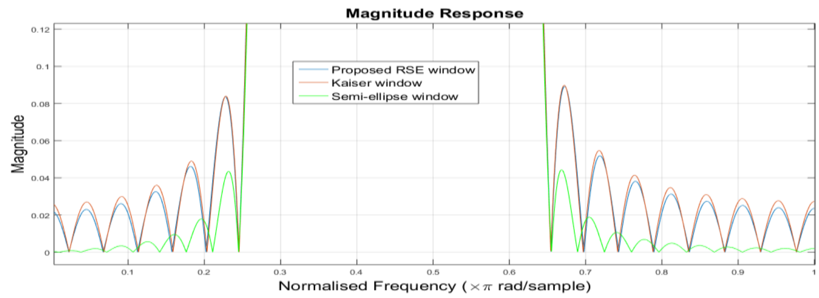
Also, $\Delta\omega = \min(0.1\pi, 0.05\pi) = 0.05\pi$ (rad/sample)

iii. Also from Table 1, calculate ω_{c_1} and ω_{c_2} of the ideal passband filter as $\omega_{c_1} = 0.275\pi$ and $\omega_{c_2} = 0.625\pi\ rad/sample$.
iv. Like in Example I, compute the parameter, β according to the A_s of the prescribed filter specification - β for RSE = 0.833 and β for Kaiser = 0.663
v. Using Eqs. (12), (16) and (13) the filter length, N required for the specified transition width are 41, 41 and 57 for RSE, Kaiser and Semi-ellipse windows, respectively.
vi. As in Example I, compute coefficients of the window function according to β and N obtained in Step 4 and Step v.
vii. Also, calculate coefficients of the ideal bandpass filter impulse response as in Table 1 using results of Step v.
viii. Finally, calculate the causal finite-duration impulse response using values obtained in Step vi and Step vii.

The plot of frequency responses of the filter using the windows for the parameters obtained were made using Matlab. The minimum stopband attenuation obtained for RSE, Kaiser, and Semi-ellipse were -21.75, -21.77 and -27.41 dB; the transistion widths achieved were 0.0424π, 0.0436π and 0.0514π rad/sample, respectively. The designed bandpass filter's magnitude responses are shown in Figure 8.



(a)



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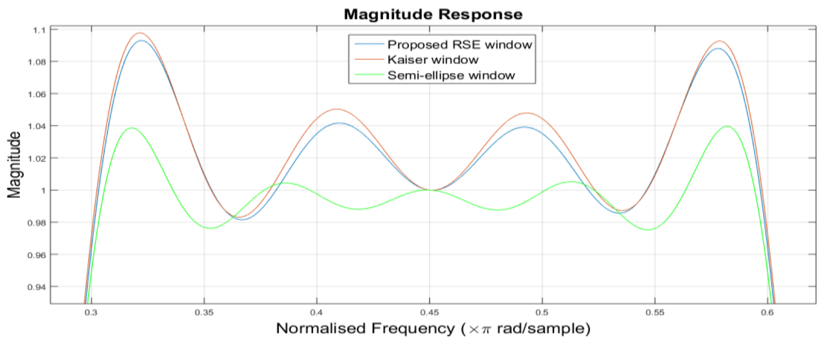

(c)

Figure 8. Magnitude to Frequency plots of Example II, (a) bandpass filters, an enlarged view of: (b) stopband attenuation and (c) passband ripples.

3. RESULTS AND DISCUSSION

The application illustrations for filter design, namely the adjustable windows – proposed RSE and Kaiser Windows with the fixed window - the Semi-ellipse window meets the specified requirements for the filters presented. The numerical results of Example I is summarized in Tables 2 for the lowpass filter while Example II is summarized in Table 3 for the bandpass filters.

Table 2. Results of lowpass filter example

Prescribed filter specification $\Delta\omega$ = **0.08π rad/sample, A_s = 23.986 dB**				
Window type	**Filter Length, N**	**Transition Width, $\Delta\omega$ (xπ rad/sample)**	**Minimum Attenuation, A_s (dB)**	**Stopband**
Proposed RSE (β = 0.502)	29	0.0783	24.55	
Kaiser (β = 1.140)	29	0.0729	24.77	
Semi-ellipse	35	0.0849	27.11	

Table 3. Results of bandpass filter example

Prescribed filter specification $\Delta\omega$ = **0.05π rad/sample, A_s = 22 dB**				
Window type	**Filter Length, N**	**Transition Width, $\Delta\omega$ (xπ rad/sample)**	**Minimum Attenuation, A_s (dB)**	**Stopband**
Proposed RSE (β = 0.663)	41	0.0436	21.77	
Kaiser (β = 0.833)	41	0.0424	21.75	
Semi-ellipse	57	0.0514	27.41	

As observed in Figure 7(a) and 8(a) and also presented in Tables 2 and 3, all the windows produced transition width that closely approximate the specified tannsition width and also the stopband attenuation. Although, the filters designed with the proposed RSE window produced better approximation as against that with the Kaiser window.

Figures 7(c) and 8(c) show that the filters implemented with the fixed Semi-ellipse window produced reduced passband ripples than the other filters implemented using the adjustable windows presented in this study. Besides, the Semi-ellipse window produced more minimum stopband attenuation of 27dB in both examples as observed in Tables 2 and 3 than the prescribed stopband attenuation of 24dB and 22dB for the lowpass filter and the bandpass filter, respectively. The choice of the Semi-ellipse window, with better stopband attenuation than the adjustable windows presented in this study, produced more filter length which compensated for the transition width as expected. This is observed in Table 2 where the number of coefficients required to implement the lowpass filters using Semi-ellipse window is 35 and 57 for the bandpass filter as in Table 3. In both Examples as seen in Tables 2 and 3, fewer coefficients are used to implement the same filters for the cases of the proposed RSE window and Kaiser window. Besides, increase in filter length increases the filter complexity which is not desired in FIR filter design.

In both Examples, the two adjustable windows – the proposed RSE window and the Kaiser window - which can be tuned with their independent parameters β to satisfy the prescribed filters in both illustrations implement the filters with the fewer filter coefficients (29 for the lowpass filter (see Table 2), and 41 for the bandpass filter (see Table 3)) than the Semi-ellipse window which indicates reduced filter complexity. Though the proposed RSE and Kaiser window produced the same filter coefficients, the proposed window produced more reduced ripples along the passband and stopband than the Kaiser window (see Figures 7 and 8, parts (b) and (c)).

4. CONCLUSION

This study proposed a new 2-parameter adjustable window which is developed from the Semi-ellipse window function. The 2 parameters are the filter length, N and parameter β. The window spectral characteristic revealed that increasing N and keeping β constant makes the main-lobe width narrower but does not affect the maximum amplitude of the side-lobe. Similarly, increasing β while keeping N constant controls the shape of the window function. The parameter β which can vary from 0 to 1 produces a Semi-ellipse window when β = 0 and a rectangular window when β = 1. In filter design, its minimum stopband attenuation increases from 21 to 27 dB as β decreases from 1 to 0 and remains relatively independent of the filter length. In filter design, the performance of the proposed RSE window was proven to produce more reduced ripples than the Kaiser window while both windows used approximately the same filter length to design a particular filter specification.

Besides, coefficients formulation of the proposed RSE window and design algorithm simplicity may make it preferable to the Kaiser window.

A new adjustable window has been introduced - the Raised semi-ellipse window. Studies were carried out on its spectral characteristic and filter applications using some illustrations. It was confirmed that its performance can be compared with Kaiser - a widely used adjustable window. It also confirms its improvement on the Semi-ellipse window function from fixed to adjustable. This will in no doubt be used in DSP to design better quality and more economical filters.

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