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Design of Robust Centralized PID Optimized LQR Controller for Temperature Control in Single-Stage Refrigeration System

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ABSTRACT

Refrigeration systems are used for many purposes such as food preservation, cooling and others. They require controller to ensure that the refrigerating cycle can go ON and OFF to maintain a setpoint temperature. For instance, in preservation of food or other perishables, deterioration can occur without efficient system to ensure that temperature within refrigerating space is kept at a setpoint value. This paper presents robust centralized proportional integral and derivative (PID) optimized linear quadratic regulator (LQR) temperature control system for single-stage refrigeration system. A composite technique in which PID algorithm was used to adjust the gains of LQR is proposed. The model of single-stage vapour compressor refrigeration (VCR) system was established in terms of the evaporator, compressor, condenser and the expansion valve's temperatures. An LQR was initially designed. Then a PID optimized LQR was design. The results indicated that the PID optimized LQR controller outperformed the LQR by providing 73.4% and 62.7% improvement for the evaporating temperature, 45.6% and 71.4% improvement for the compression temperature, 30% and 84.6% improvement for the condensing temperature, and lastly 72% and 70.2% improvement for the expansion temperature in terms of response time and settling time. Simulation with test data proved its robustness and effectiveness in tracking setpoint temperature. Generally, the proposed system has shown capacity to offer robust and centralized tracking in the presence of changing setpoint values.

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1. INTRODUCTION

Refrigeration industry plays key and increasing role, with meaningfully contribution made in food, health, energy, and environment sectors in today's global economy [1]. Refrigeration systems need operating controls so that the cycle can go ON and OFF to keep a fixed (setpoint) temperature.

It is one of the largest energy consumers in a facility. There are certain challenges in refrigeration systems such as proper control of superheat for efficient and safe operation of the system, and also maintaining the temperature of the refrigerated space or foodstuff within the desired requirement. For example, to prevent possible deterioration of foodstuff in refrigeration system efficient strategy is required to maintain the temperature within the space. The use of ON and OFF thermostat or relay based control drive the system temperature on limited cycle. The time duration and magnitude of such cycle affects a number of relevant characteristics of the system.

Refrigeration systems are generally designed for predetermined capacity to achieve cooling capacity in terms of the maximum demand at the highest ambient temperature. Conventionally, it is designed with an ON and OFF control so as to be able to adjust the cooling demand [2]. In recent times, different control schemes have been well developed to improve the effectiveness and reliability of the refrigeration systems so as to regulate its internal temperature more effectively and accurately. The control strategy used in refrigeration system is an essential device in terms of thermal performance [3].

On the need to enhance the temperature regulation of refrigeration system, several control techniques have been used. The proportional integral and derivative (PID) control technique is said to be the most famously used in the industry due to its precision, feasibility and simplicity [4]. Nevertheless, many other techniques that can solve the problem of nonlinearity, error in system model, provide the knowledge of the internal variable of the system, and offer optimal steady state error compared to PID are being proposed in literature [5]. For instance, [6] addressed the problem of controlling the outlet temperature of a one-stage refrigeration system using decentralized control technique by designing Generalized- Proportional-Integral (GPI) observers for the internal loops of each controlled variable while PID controllers were tuned based on the Quantitative Feedback Theory (QFT) for the individual external loops. Decentralized linear active disturbance rejection control (LADRC) technique applied in benchmark refrigeration system was achieved by manually tuning two second-order LADRCs without the model of the system [7]. A modified classical PID controller was designed by the application of special method called Han' nonlinear PID (NPID) control system, was used in [8] for controlling the temperature of the evaporator and the degree of superheating through the compressor speed and the expansion valve opening. Fuzzy logic controller (FLC) was applied to a third-order discrete state space system for temperature control inside refrigeration, which computes the internal air temperature according to the temperature at the level of the evaporator exchanger [9]. A multi-objective optimization design (MOOD) for the control of vapour compression of refrigeration system was achieved in [10]. Model Predictive Control (MPC) based controller using Recursive Integration Optimal Trajectory Solver (RIOTS) for the control of one-staged refrigeration cycle model was presented in [11]. Also, modeling and control of industrial refrigeration system for freezing food that aimed to minimize total energy consumption in addition to utilizing the maximum amount of renewable energy was achieved using MPC [12]. A technique utilizing model-based feedback controller and a learning feed-forward algorithm was used to control a refrigeration system in [13]. PID controller was used to control the outlet temperature of the evaporator of a refrigeration system [14]. A method to tune two decentralized PID controllers based on the competition over resources (COR) metaheuristic optimization algorithm, which is a multi-objective optimization design, was used to solve the problem of vapour compression refrigeration system [15]. The temperature of a variable speed compression (VSC) refrigeration system was maintained at defined value of 5^{o}C using FLC in [16].

With several control techniques introduced to regulate the temperature of refrigeration system, each has its performance limitation. The PID controller was shown to be robust to changes in setpoint temperature of evaporator but was not able to deal with a small input disturbance and was sensitive to changes in parameters and as such resulting in instability [14]. Despite the fact that LQG controllers provide not only optimal control but also are able to estimate non-measurable states, their performances depend on model and parameter accuracy of the system, which is the same for linear quadratic regulator (LQR). The major challenge in MPC even though it is considered a foremost choice in addressing many complex control problems is nonlinearity in plant model [17]. FLC suffers from steady state error [18]. For instance in [16], FLC maintain a setpoint temperature for refrigeration system with steady-state error and as such its control variable is usually augmented with gain or compensator as in [19,20]. Also, most of the control techniques have been limited to the control of a particular temperature such as outlet temperature of the evaporator, which may result in implementing different (or decentralized) controllers for a refrigeration system and thereby increasing design complexity and implementation cost.

Now, taking into account the high parameter variability of refrigeration system, a predetermined feedback control approach based on composite model may serve well as a solution if fixed requirements on food preservation and energy consumption by a refrigeration system are to be achieved. Also, one critical performance parameters of VCR system is its pull-down time (PDT), which is a measure of the time taken for evaporator to attain its lowest temperature under various environmental conditions [21]. Thus, the time taken for the VCR system to reach its optimum cooling control is based on the PDT. Hence, it is expected that a control system will ensure that the refrigeration system reaches its optimum cooling performance with respect to a setpoint temperature of the evaporator as soon as possible at reduced PDT. In this paper, a composite centralized scheme that involves the use of PID parameters to optimize a LQR for temperature control in the components of single-stage vapour compressor refrigeration (VCR) system is proposed. The effectiveness of the proposed control system is basically focused on the ability to ensure robustness to setpoint evaporator temperature control with reduced PDT.

2. RESEARCH METHOD

This section is divided into three subsections. It presents the mathematical expressions for the working principle of the VCR system. Subsequently, the data for various parameters of the refrigeration system were computed. A linear quardratic regulator (LQR) was initially designed for temperature control. MATLAB was used to compute the data for the various parameters of the LQR controller. Finally, the proposed controller is designed.

2.1. Modelling of Single-Stage Refrigeration System

The dynamic equations of single-stage vapour-compressor refrigeration (VCR) system are established using lumped parameter [22], in this subsection. The VCR system consists of four components: evaporator, compressor, condenser, and expansion valve as shown by the block diagram in Figure 1. Mathematical models are formulated assuming the refrigeration cycle is a closed process, operation of VCR system is in steady state, the refrigerant flowing through the various units has the same mass flow rate, flow is continuous, there is no variation over time regarding the refrigerant's properties at any given point in the system, and all variables along the control volume are uniform. That is, there are no significant variations in variables such as kinetic energy and potential energy along the finite volume. Also, thermal equilibrium exists between the refrigerant liquid and vapour phases, the heat exchanger has perfect thermal insulations, and the pipes have negligible axial heat conditions [22].

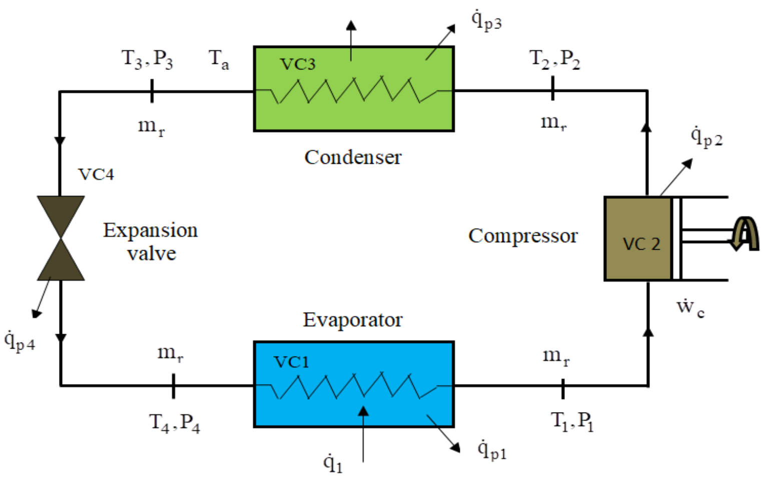

Figure 1. Block diagram of refrigeration system

For the refrigeration system in Fig. 1, the equations regarding the principle of energy conservation are defined by [22]:

$$\dot{q} + \dot{w} = \dot{m}\left[(h_f - h_o) + \frac{v_f^2 - v_o^2}{2} + g(z_f - z_o)\right] \quad (1)$$

$$\dot{q} + \dot{w} + \dot{q}_{pi} = \dot{m}_r c_{pi}(T_f - T_o) \quad (2)$$

$$q_{pi} = C_{Ti}\frac{dT_i}{dt} + \frac{(T_i - T_a)}{RT_i} \quad (3)$$

where $\dot{q}$ and $\dot{w}$ are the quantity of heat transferred from the surrounding heat flow to system and the compressor's electric motor work done. $\dot{m}$ is mass flow rate, $h = (h_f - h_o)$ is the enthalpy function of initial and final enthalpy, $(v_f^2 - v_o^2)/2$ and $g(z_f - z_o)$ are the change in linear kinetic energy $(v^2/2)$ and potential energy (gz), g is the gravitational acceleration, z is height, and v is velocity. $\dot{m}_r$ is mass flow rate of circulating refrigerant, c_{pi} stands for the heat capacity per unit mass at room temperature, $(T_f - T_o)$ is change in temperature, $\dot{q}_{pi}$ equals heat generated or loss and is defined by the cooling law of Newton in (3) [22,23]. C_{Ti} is the heat capacity of the evaporator component confined interior space and for other components , T_i the temperature inside the system, T_a is the room (or ambient) temperature, and dT_i/dt is the cooling speed in the interior. The subscripts o and f represent initial and final states. Simplification of (1) assuming variation in kinetic energy and potential energy resulted in (2) [22].

The mathematical models of the evaporator, compressor, condenser, and expansion valve are established in (4) to (7) [22]:

$$\frac{dT_1}{dt} = T_1\left(\frac{c_p\dot{m}_r}{C_{T1}} - \frac{1}{C_{T1}R_{T1}}\right) - T_4\frac{c_p\dot{m}_r}{C_{T1}} + \frac{T_a}{C_{T1}R_{T1}} - \frac{\dot{q}_1}{C_{T1}} \tag{4}$$

$$\frac{dT_2}{dt} = -T_1\frac{c_p\dot{m}_r}{C_{T2}} + T_2\left(\frac{c_p\dot{m}_r}{C_{T2}} - \frac{1}{C_{T2}R_{T2}}\right) + \frac{T_a}{C_{T2}R_{T2}} - \frac{\dot{w}_c}{C_{T2}} \tag{5}$$

$$\frac{dT_3}{dt} = -T_2\frac{c_p\dot{m}_r}{C_{T3}} + T_3\left(\frac{c_p\dot{m}_r}{C_{T3}} - \frac{1}{C_{T3}R_{T3}}\right) + \frac{T_a}{C_{T3}R_{T3}} - \frac{\dot{q}_3}{C_{T3}} \tag{6}$$

$$\frac{dT_4}{dt} = -T_3\frac{c_p\dot{m}_r}{C_{T4}} + T_4\left(\frac{c_p\dot{m}_r}{C_{T4}} - \frac{1}{C_{T4}R_{T4}}\right) + \frac{T_a}{C_{T4}R_{T4}} \tag{7}$$

From (4) to (7), c_{pi} in (2) is replaced with c_p, $\dot{q}_1$ and $\dot{q}_3$ are the evaporator heat rate and condenser heat rate, R_{Ti} is replaced with R_{T1}, R_{T2}, R_{T3}, and R_{T4}, and $i = 1, 2, 3, 4$ is the ith component of the refrigeration system with respect to evaporator, compressor, condenser, and expansion valve and T_1, T_2, T_3, and T_4 are the corresponding temperatures.

The mathematical equations of the various units of the VCR system defined in (4) to (7) are first order differential equations given in terms of their individual output temperature. Thus, these equations can be defined in matrix form by [22]:

$$\begin{bmatrix}\dot{T}_1\\ \dot{T}_2\\ \dot{T}_3\\ \dot{T}_4\end{bmatrix} = \begin{bmatrix}\left(\frac{c_{p1}\dot{m}_r}{C_{T1}} - \frac{1}{C_{T1}R_{T1}}\right) & 0 & 0 & -\frac{c_{p1}\dot{m}_r}{C_{T1}}\\ -\frac{c_{p2}\dot{m}_r}{C_{T2}} & \left(\frac{c_{p2}\dot{m}_r}{C_{T2}} - \frac{1}{C_{T2}R_{T2}}\right) & 0 & 0\\ 0 & -\frac{c_{p3}\dot{m}_r}{C_{T3}} & \left(\frac{c_{p3}\dot{m}_r}{C_{T3}} - \frac{1}{C_{T3}R_{T3}}\right) & 0\\ 0 & 0 & -\frac{c_{p4}\dot{m}_r}{C_{T4}} & \left(\frac{c_{p4}\dot{m}_r}{C_{T4}} - \frac{1}{C_{T4}R_{T4}}\right)\end{bmatrix}\begin{bmatrix}T_1\\ T_2\\ T_3\\ T_4\end{bmatrix} + \begin{bmatrix}\frac{1}{C_{T1}R_{T1}} & -\frac{1}{C_{T1}} & 0 & 0\\ \frac{1}{C_{T2}R_{T2}} & 0 & -\frac{1}{C_{T2}} & 0\\ \frac{1}{C_{T3}R_{T3}} & 0 & 0 & -\frac{1}{C_{T3}}\\ \frac{1}{C_{T4}R_{T4}} & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}T_a\\ \dot{q}_1\\ \dot{w}_c\\ \dot{q}_3\end{bmatrix} \tag{8}$$

$$T(t) = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}T_1\\ T_2\\ T_3\\ T_4\end{bmatrix} \tag{9}$$

The matrix equations in (8) and (9) can be represented by the general state space expression defined by [24]:

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) \tag{10}$$

$$y(t) = \mathbf{C}x(t) + \mathbf{D}u(t) \tag{11}$$

where $\dot{x}(t) = \dot{T}$, $x(t) = [T_1 \quad T_2 \quad T_3 \quad T_4]^T$, $u(t) = [T_a \quad \dot{q}_1 \quad \dot{w}_c \quad \dot{q}_3]^T$, and $y(t) = T$. Substituting the values of the parameters of the VCR system in Table 1 into (8) and (9) gives the following in terms of the state matrix **A**, input matrix **B**, output matrix **C**, and the transition matrix **D**:

$$\mathbf{A} = \begin{bmatrix} -0.0024 & 0 & 0 & -0.0000103 \\ -0.0000196 & -0.0160 & 0 & 0 \\ 0 & -0.00003186 & -0.0167 & 0 \\ 0 & 0 & -0.0000923 & 0.0005327 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} -0.00247 & 0.000222 & 0 & 0 \\ 0.016 & 0 & -0.0004 & 0 \\ 0.0167 & 0 & 0 & -0.0008 \\ 0.000625 & 0 & 0 & 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \boldsymbol{D} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Table 1. Parameters of the VCR system [22]

Description	Symbol	value	Unit
Specific heat of refrigerant (evaporator)	C_{p1}	1330	J/kg-k
Specific heat of refrigerant (compressor)	C_{p2}	1400	J/kg-k
Specific heat of refrigerant (condenser)	C_{p3}	1138	J/kg-k
Specific heat of refrigerant (expansion valve)	C_{p4}	1318	J/kg-k
Thermal capacity (evaporator)	C_{T1}	4500	J/K
Thermal capacity (compressor)	C_{T2}	2500	J/K
Thermal capacity (condenser)	C_{T3}	1250	J/K
Thermal capacity (expansion valve)	C_{T4}	500	J/K
Mass flow rate	$\dot{m}_r$	0.000035	Kg/s
Thermal resistance (evaporator)	R_{T1}	0.090	K/W
Thermal resistance (compressor)	R_{T2}	0.025	K/W
Thermal resistance (condenser)	R_{T3}	0.048	K/W
Thermal resistance (expansion valve)	R_{T4}	3.20	K/W
Evaporator heat rate	$\dot{q}_1$	195	W
Condenser heat rate	$\dot{q}_3$	-200	W
Ambient	T_q	298	K
Compressor power input	$T_{R,}\ W_c$	5	W

2.2. Proposed Control System Design

The proposed temperature control system for single-stage VCR is shown in Figure 2. The control algorithm of LQR is optimized using the PID controller. The objective is to achieve a centralized control of temperature in various units of a VCR refrigeration system with reduced rise (response) time, settling time, overshoot, steady state error, and improved tracking performance. The measurement device is assumed to be a unity feedback gain sensor.

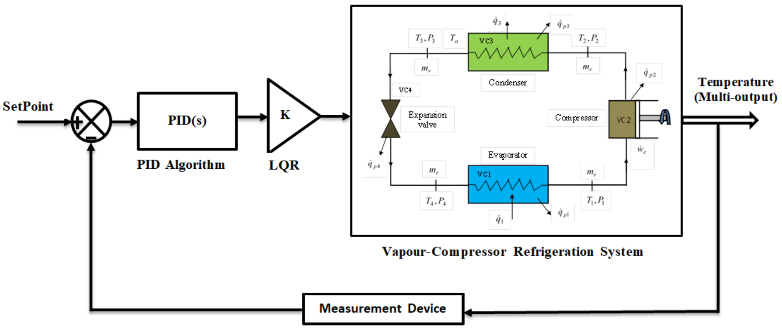

Figure 2. Block diagram of proposed system

The composite control scheme is such that each unit of the single-stage refrigeration system is separately connected to the PID-LQR controller while it maintains the same parameter and configuration. Hence it is considered a centralized control system because the controller simultaneously serving all the units in the refrigeration system is one and the same. The MATLAB/Simulink model is shown in Figure 3.

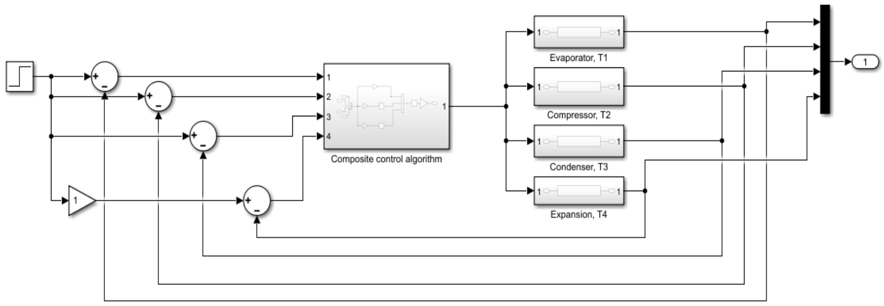

Figure 3. Simulink model of the proposed system

2.3. Design of Linear Quadratic Regulator

In modern optimal control theory, the LQR design technique is used and it is commonly applied in several control system just like the PID controller as a result of its stability [25]. Designing LQR involves the application of state-space technique to analyze and control a linear time invariant (LTI) system such as the established VCR system model. Therefore, to design the LQR it is assumed that the model of the VCR system is perfectly known and all states are available for feedback [26].

The design of LQR as optimal state feedback control law is aimed at minimizing the quadratic cost function defined by:

$$J = \frac{1}{2}\int_{t_0}^{t_f}\left(x^T(t)Qx(t) + u^T(t)Ru(t)\right)dt \tag{12}$$

where Q and R are individual weighting matrix (or factor) of states and control variables respectively. The expression (12) ensures that optimal response is provided by the controller.

Generally, three steps are involved while designing LQR. The first step deals with selecting the appropriate values for the state and control law matrices Q and R. This is usually achieved during design by repeated iteration wherein Q is varied and R is fixed. In this paper, the values of Q and R were determined in MATLAB using appropriate syntax. For the second step, the algebraic Riccati equation (ARE) is computed. Lastly, the optimal gain matrix K of the control law u is determined. Thus, the following equations are used to determine the Riccati matrix P, K, and u.

$$PA + A^TP - P - PBR^{-1}B^T + Q = 0 \tag{13}$$

$$K = R^{-1}B^TP \tag{14}$$

$$u = -Kx \tag{15}$$

The computed values of P and K using (13) and (14) after several iterations for Q at fixed value of R for optimal response using the MATLAB syntax $[K, S, P] = \mathrm{lqr}(A, B, Q, R)$ are:

$$Q = \begin{bmatrix} 10000 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 \\ 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 10000 \end{bmatrix}, R = [1], P = \begin{bmatrix} -0.0015 \\ -0.0229 \\ -0.0647 \\ -2.3277 \end{bmatrix}$$

$$K = \begin{bmatrix} 16.3295 & 68.2268 & 69.5506 & 26.4429 \\ -86.2227 & 8.0669 & 4.1615 & 17.2451 \\ 14.5349 & -29.5418 & 22.6250 & 50.6223 \\ 14.9963 & 45.2501 & -50.2648 & 36.3840 \end{bmatrix}$$

Therefore, the designed optimal control law for the LQR with respect to temperature control in the considered single-stage VCR system is given by (16) by substituting the values of K into (15). The Simulink model of the LQR is shown in Figure 4.

$$u = -\begin{bmatrix} 16.3295 & 68.2268 & 69.5506 & 26.4429 \\ -86.2227 & 8.0669 & 4.1615 & 17.2451 \\ 14.5349 & -29.5418 & 22.6250 & 50.6223 \\ 14.9963 & 45.2501 & -50.2648 & 36.3840 \end{bmatrix} x \tag{16}$$

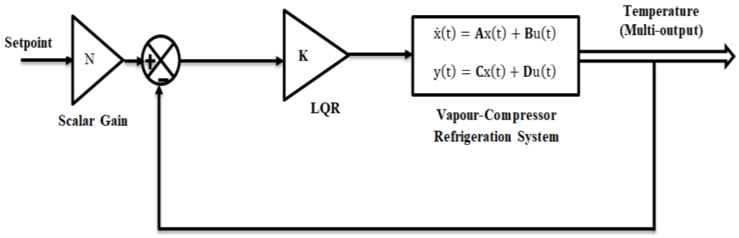

Figure 4. LQR based temperature control system for single-stage VCR

2.4. PID Optimized Linear Quadratic Regulator

This subsection is based on optimizing the gain K of the LQR by enhancing its performance via the tuning of the nparameters of PID controller. The computational process of the PID algorithm involving the summing of simultaneous actions of the proportional, integral and differentiation techniques in (17) is initially used to compensate for the resulting error between setpoint (or desired) temperature and actual temperature of an ith component of the refrigeration system.

The action of the PID controller is mathematically defined for a bounded control by:

$$u_{pid}(t) = k_p e(t) + k_i \int_{t_0}^{t_f} e(t)dt + k_d \frac{de(t)}{dt} \quad (17)$$

where k_p, k_i and k_d are the parameters of PID called proportional gain, integral gain and derivative gain, and e(t) is the error signal represented in terms of deviation between desired setpoint temperature T_{di} and the actual temperature T_i of the ith component of the VCR system: $e(t) = (T_{di} - T_i)$ and substituting this into (17) gives:

$$u_{pid}(t) = k_p(T_{di} - T_i) + k_i \int_{t_0}^{t_f}(T_{di} - T_i)dt + k_d \frac{d(T_{di}-T_i)}{dt} \quad (18)$$

Since T_{di} is a constant, its derivative is zero and as such the PID control law in (18) can be defined by:

$$u_{pid}(t) = k_p(T_{di} - T_i) - k_d \frac{dT_i}{dt} + k_i \int_{t_0}^{t_f}(T_{di} - T_i)dt \quad (19)$$

Now, the gain K of the LQR is defined by the $n \times n$ matrix in (20). Assigning K_i for $i = 1, 2, 3, 4$ to represent each row in (20) results in $n \times m = K_i \times 1$ and this gives the matrix in (21).

$$K = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \quad (20)$$

$$K = [K_1 \quad K_2 \quad K_3 \quad K_4]^T \quad (21)$$

Using the PID controller to adjust the gain K of the LQR produces a control action u_{pid} that drives the gains of the LQR into new control law that eventually resulted in a control command action that propels the system response into new state defined by:

$$u_{pid-lqr} = u_{pid} \times (K_i \times 1) = \begin{bmatrix} k_p K_1(T_{d1} - T_1) - k_d K_1 \frac{dT_1}{dt} + k_i K_1 \int_{t_0}^{t_f}(T_{d1} - T_1)dt \\ k_p K_2(T_{d2} - T_2) - k_d K_2 \frac{dT_2}{dt} + k_i K_1 \int_{t_0}^{t_f}(T_{d2} - T_2)dt \\ k_p K_3(T_{d3} - T_3) - k_d K_3 \frac{dT_3}{dt} + k_i K_3 \int_{t_0}^{t_f}(T_{d3} - T_3)dt \\ k_p K_4(T_{d4} - T_4) - k_d K_4 \frac{dT_4}{dt} + k_i K_4 \int_{t_0}^{t_f}(T_{d4} - T_4)dt \end{bmatrix} \quad (22)$$

where $T_{d1} - T_1 = e_1$, $\dot{e_1} = \dot{T_1}$, $T_{d2} - T_2 = e_2$, $\dot{e_2} = \dot{T_2}$, $T_{d3} - T_3 = e_3$, $\dot{e_3} = \dot{T_3}$, $T_{d4} - T_4 = e_4$, $\dot{e_4} = \dot{T_4}$.

Since $K = K_i \times 1$ corresponds to $n \times m$ (i.e. n = 4 and m =1) as in (21) and is constant and also the same for the control of the various ith components of the refrigeration system, hence (22) can be further simplified by the following expressions given by:

$$u_{\mathrm{pid-lqr}} = \begin{bmatrix} \mathrm{k_p}K_1\mathrm{e}_1 - \mathrm{k_d}K_1\,\dot{\mathrm{e}_1} + \mathrm{k_i}K_1\int_{\mathrm{t_0}}^{\mathrm{t_f}}\mathrm{e}_1\mathrm{dt} \\ \mathrm{k_p}K_2\mathrm{e}_2 - \mathrm{k_d}K_2\dot{\mathrm{e}_2} + \mathrm{k_i}K_1\int_{\mathrm{t_0}}^{\mathrm{t_f}}\mathrm{e}_2\mathrm{dt} \\ \mathrm{k_p}K_3\mathrm{e}_3 - \mathrm{k_d}K_3\dot{\mathrm{e}_3} + \mathrm{k_i}K_3\int_{\mathrm{t_0}}^{\mathrm{t_f}}\mathrm{e}_3\mathrm{dt} \\ \mathrm{k_p}K_4\mathrm{e}_4 - \mathrm{k_d}K_4\dot{\mathrm{e}_4} + \mathrm{k_i}K_4\int_{\mathrm{t_0}}^{\mathrm{t_f}}\mathrm{e}_4\mathrm{dt} \end{bmatrix} \tag{23}$$

Or,

$$\mathrm{u}_{pid-lqr} = \mathrm{u}_{pid}\mathrm{K} = \mathrm{k_{pk}}e_i - \mathrm{k_{dk}}\dot{e_i} + \mathrm{k_{ik}}\int_{\mathrm{t_0}}^{\mathrm{t_f}} e_i\mathrm{dt} \tag{24}$$

Therefore the gains of the PID optimized LQR control system for VCR are $\mathrm{k_p K} = \mathrm{k_{pk}}$, $\mathrm{k_i K} = \mathrm{k_{ik}}$ and $\mathrm{k_d K} = \mathrm{k_{dk}}$. This is assigned or expressed in terms of the PID gains because they are the parameters that can be adjusted over time in the designed controller. The values of the PID parameters are: $\mathrm{k_p} = 50.00$, $\mathrm{k_i} = 0.10$, $\mathrm{k_d} = 2.00$.

It should be note that despite the fact that the PID controller is a three-term control algorithm, in depth and thorough mathematical induction was used to come up with the application of the three-term PID algorithm to optimize or tune the four-gain matrix or (4×1) matrix of the LQR control law. This was achieved by applying mathematical methods based on differential and matrix theorems. This approach was explicitly presented. Though the PID-LQR has been used in many previous control systems, areas and systems where it has been applied differs. The PID-LQR presented in this paper is unique in the sense that it simultaneously provided control for various elements of single-stage refrigeration system. Also, in most design, the LQR is used to optimize the PID parameters but in this, case the PID is used to optimize the LQR gains instead. This is unlike previous approaches that have mainly been used as decentralized methods.

3. RESULTS AND DISCUSSION

This section presents the results of the simulation analysis carried out in MATLAB/Simulink environment with respect to unit step input response to ascertain the effectiveness of the proposed system. Simulations were conducted in accordance to the test results in [22] for evaporating temperature, compression temperature, condensing temperature and the expansion temperature which are approximately equal to 265 K, 317 K, 315 K, and 294 K. Also, with refrigerant R134a, evaporating temperature and the condensing temperature from the validation test of VCR system were 255 K and 335 [22] and these were simulated to further check the effectiveness of the proposed system.

3.1. Step Response of LQR and PID-LQR

The response of the simulation test with respect to unit step temperature input is presented in this subsection for the designed LQR and the PID optimized LQR control systems. The resulting step response plots for LQR and PID-LQR are shown in Figure 5 and Figure 6. The comparison plots of both controllers are presented in Figure 7. The time domain characteristics performances of the units of the VCR system controlled by LQR and PID-LQR are shown in Tables 2 and 3 respectively.

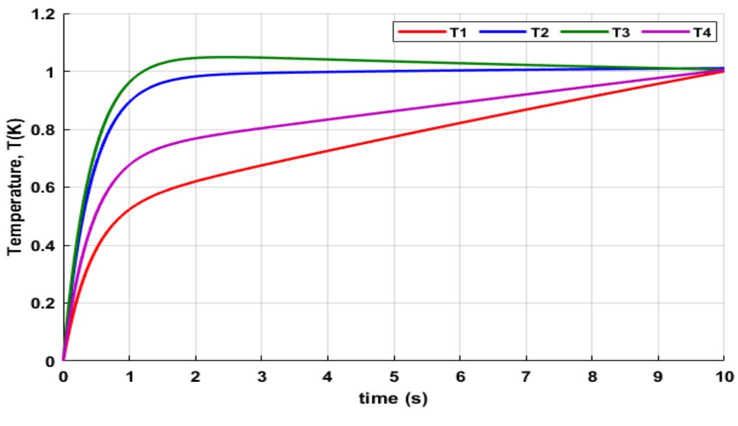

Figure 5. Step responses of LQR controlled VCR system

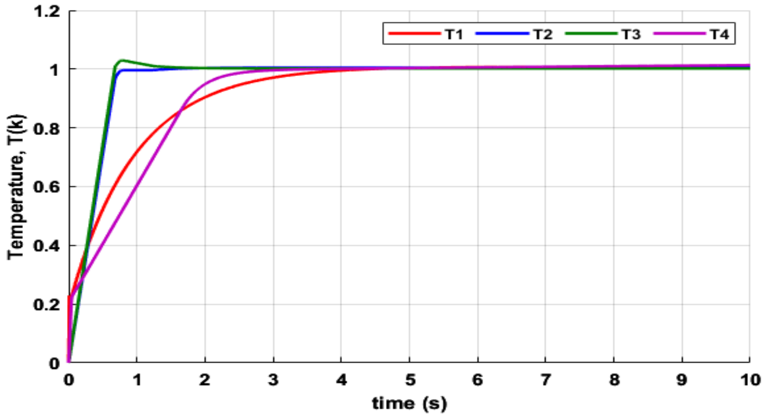

Figure 6. Step response of PID-LQR controlled VCR system

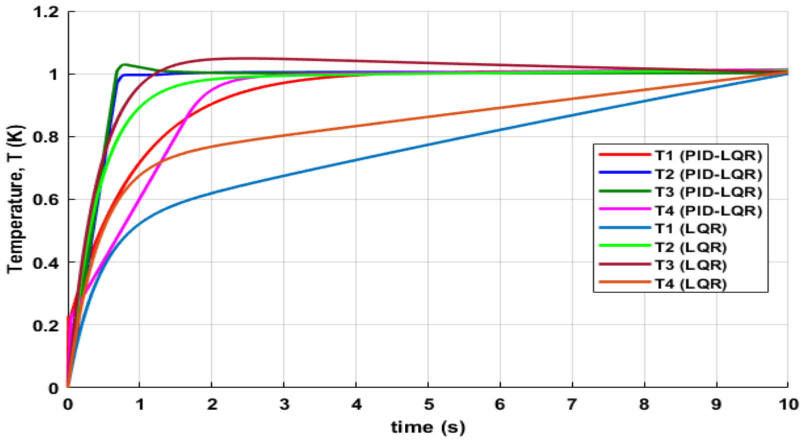

Figure 7. Step response of PID-LQR controlled VCR system

Table 2. Numerical analysis of LQR control system response to unit step input

Temperature	Time Domain Parameters			
	Rise time (s)	Settling time (s)	Overshoot (%)	Steady state error
Evaporating (T_1)	7.63	9.53	0	0
Compression (T_2)	1.03	2.52	0	0
Condensing (T_3)	0.77	6.29	4.25	0.048
Expansion (T_4)	6.39	9.28	0	0

Table 3. Numerical analysis of PID-LQR control system response to unit step input

Temperature	Time Domain Parameters			
	Rise time (s)	Settling time (s)	Overshoot (%)	Steady state error
Evaporating (T_1)	2.03	3.55	0	0
Compression (T_2)	0.56	0.72	0	0
Condensing (T_3)	0.54	0.97	2.72	0.029
Expansion (T_4)	1.79	2.77	0	0

Looking at Figure 5 and Table 2, it can be seen that the LQR controller was able to achieve the desired temperature at the expense of high rise (or response) time and settling time especially for the evaporating temperature and expansion temperature. Hence, the LQR controller was enhanced with PID algorithm. The simulation plots and table of the PID optimized LQR control system performance for the VCR in Figure 6 and Table 3 indicated that improved tracking performance was achieved compared to the conventional LQR control system.

It can be deduced looking at Figure 7 and Tables 3 and 2 that the proposed PID optimized LQR provided better performance over the conventional LQR in terms of improved tracking for all units temperature in the system together with response time and settling time that amounted to: 73.4% and 62.7% improvement for the evaporating temperature, 45.6% and 71.4% improvement for the compression temperature, 30% and 84.6% improvement for the condensing temperature, and lastly 72% and 70.2% improvement for the expansion temperature. Generally, the proposed system has proven to be robust and able to provide centralized control for all the various elements of the VCR with fast response time and overall improved transient and steady-state characteristics.

3.2. Step Response Validation Analysis

The effectiveness of the proposed system is validated in this subsection via step response simulation analysis using test results of the temperature of the various units of the VCR system including the validation test result with refrigerant R134a for the evaporating temperature and condensing temperature. The step response plots of temperature for the various units considering their various test result temperatures are shown in Figure 8. The evaporating temperature and the condensing temperature responses with R134a are shown in Figure 9.

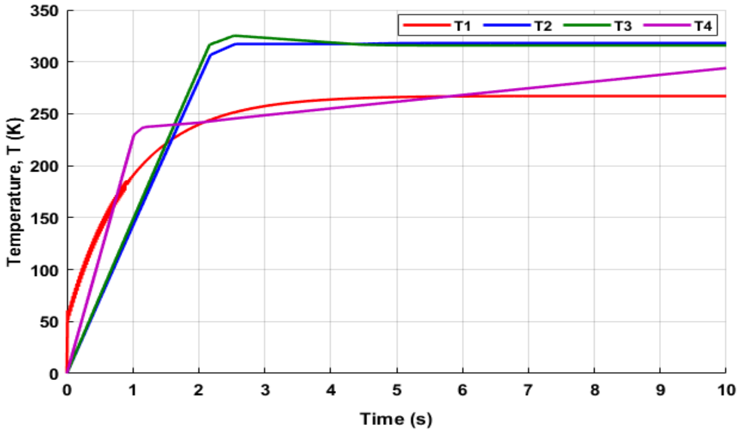

Figure 8. Temperature of PID-LQR controlled VCR system

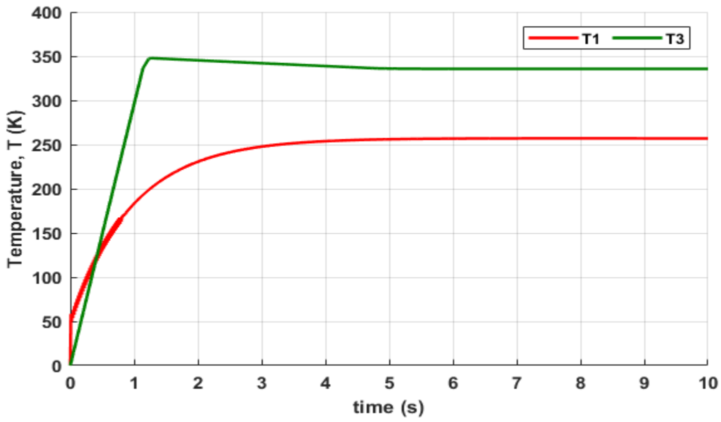

Figure 9. Temperature of PID-LQR controlled VCR system (with R134a)

Table 4. Analysis of PID-LQR control system response for given test temperature

Temperature (K)	Time Domain Parameters			
	Rise time (s)	Settling time (s)	Overshoot (%)	Steady state error
Evaporating (T_1 = 265)	2.02	3.55	0	0
Compression (T_2 = 317)	1.81	2.36	0	0
Condensing (T_3 = 315)	0.86	2.75	3.60	0.5
Expansion (T_4 = 294)	5.33	9.09	0	0

Table 5. PID-LQR control system response for given test temperature (with R134a)

Temperature (K)	Time Domain Parameters			
	Rise time (s)	Settling time (s)	Overshoot (%)	Steady state error
Evaporating (T_1 = 255)	2.02	3.55	0	0
Condensing (T_3 = 335)	0.91	2.91	3.60	0.6

The simulation results presented in Figure 8 and Table 4 indicated that the proposed control system was able to achieve the desired setpoint temperature with good transient and steady-state performance characteristics. Though the response performance of the expansion temperature seems to deteriorate in terms of response time and settling time, the system was still able to reach the desired setpoint and thereby achieved zero steady-state error. In Figure 9 and Table 5, with refrigerant R134a, the PID optimized LQR control system achieved the setpoint temperature. Generally, from the simulation analysis in Tables 4 and 5, it can be clearly seen that the proposed control system maintained the same transient and steady-state characteristics almost in all cases but achieved exact same value for the evaporating temperature. This performance proves the robustness of the proposed controller for temperature control in single-stage VCR system.

4. CONCLUSION

This paper has designed a robust PID based LQR controller whose operation is centralized to ensure that setpoint tracking of temperature for all units (evaporator, compressor, condenser, and expansion valve) is achieved simultaneously from one control location i.e. the use of the same control algorithm for all units. The effectiveness and robustness of the proposed control system to provide simultaneous setpoint tracking of temperature in each element of the single-stage VCR system was demonstrated via step response simulation test in MATLAB/Simulink. The results obtained proved its ability to achieve the design objective. Further simulations were conducted using test data of the evaporating temperature, compression temperature, condensing temperature and the expansion temperature in addition to the temperatures of the evaporating and condensing units with refrigerant R134a. The results obtained further validated the effectiveness of the proposed controller to provide robust setpoint tracking control in the presence of changing setpoint parameters especially for the evaporating temperature, compression temperature and condensing temperature. Generally, considering the effectiveness of the proposed system performance with respect to the evaporator, it can be seen that it provided robust and optimum cooling control for the refrigeration system as setpoint evaporating temperature is reached at reduced response time (2.02 s) and settling time (3.55 s) at the same value in all cases of varied temperatures. Therefore, since the PDT is measured in terms of the time taken for the evaporator to reach its optimum (or desired) temperature, it sufficed to say that the proposed system ensured that best cooling impact was achieved at reduced PDT (settling time) of 3.55 s. The PDT is taken as settling time because it is the time at which the evaporator temeperature reaches a certain predetermined lowest level and this is the same as settling time, which is the time for the response of a control system to reach acceptable steady or defined level with respect to setpoint value. However, further investigation can still be done regarding tuning the parameters of the PID controller for improved setpoint tracking for the expansion valve. Generally, the use of the proposed system will provide optimal/robust control for the single-stage VCR system. It will also provide reduced design and implementation cost. This is because rather implementing different control units, a centralized controller will be used to address the problem of temperature variations in various components of the refrigeration system. Despite the fact that the combination of PID and LQR yielded significant results, both techniques are a type of control method whose performance can be affected by variation in system parameters. Thus, future study should seek to use adaptive or intelligent based control system together with LQR. Another, limitation that can be addressed in future study is to carry experimental analysis of the designed PID-LQR controller to further validate the effectiveness of the system.

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REFERENCES

[1] D. Coulomb, “The role of refrigeration in the global economy,” International Institute of Refrigeration (IIR), 29th Informatory Note on Refrigeration Technologies, November 2015. Available: ainttrofee.nl/wp-content/uploads/2019/01/NoteTech_29-World-Statistics.pdf

[2] T. S. S. Dantas, I. c. Franco, A. M. F. Fileti, and F. V. Silva, “Dynamic linear modeling of a refrigeration process with electronic expansion valve actuator*,” International Journal of Refrigeration*, vol. 75, pp. 311-321, 2017.

[3] E. P. B. Filho, F. E. M. Garcia, and O. S. H. Mendoza, “Application of adaptive control in a refrigeration system to improve performance,” *Journal of the Brazilian Society of Mechnanical Sciences and Engineering*, vol. 33, no. 2, pp. 176-182, 2011

[4] H. Moradi, H. Setayesh, and A. Alasty, “PID-Fuzzy control of air handling units in the presences of uncertainty, “*International Journal of Thermal Sciences*, vol. 109, pp. 123-135, 2016.

[5] R. B. Ali, M. A. Jaballah, E. Aridhi, and A. Mami, “Design and FPGA-implementation of a PID controller for temperature control in a refrigeration system, *Indian Journal of Science and Technology*, vol. 11, no. 16, pp. 1-14, 2018.

[6] J. J. Carreño-Zagarra, R. Villamizar, J. C. Moreno, and J. L. Guzmán, “Active Disturbance Rejection and PID Control of a One-stage Refrigeration Cycle,” *IFAC-PapersOnline*, vol. 51, no. 1, pp. 444-449, 2018.
[7] B. Zhang, W. Tan, J. Li, and R. Zhou, “Decentralized active disturbance rejection control for benchmark refrigeration system, “*IFAC-PapersOnline*, vol. 51, no. 4, pp. 503-508, 2018.
[8] Z. Lei and Y. Zhou, “A kind of nonlinear PID controler for refrigeration systems based on vapour compression,” *IFAC-PaperOnline*, vol. 51, no. 4, pp. 716-721, 2018.
[9] J. Arfaoui, E. Feki, and A. Mami, “PID and fuzzy logic optimized controller for temperature control in a cavity of refrigeration," *IREC2015 The Sixth International Renewable Energy Congress*, 2015, pp. 1-6.
[10] G. Reynoso-Meza, H. S. Sánchez, and V. H. A. Ribeiro, “Control of refrigeration systems based on vapour compression using multi-objective optimization techniques,” “*IFAC-PaperOnline*, vol. 51, no. 4, pp. 722-727, 2018.
[11] S. Dehghan, T. Zhao, Y. Chen, Y. Zhao, J. Yuan, A. Ates, and Y-Q. Chen, “PID2018 benchmark challenge: model predictive control with conditional integral controlusing a general purpose optimal control problem solver – RIOTS,” *IFAC-PaperOnline*, vol. 51, no. 4, pp. 882-887, 2018.
[12] E. Di Mattia, A. Gambarotta, E. Marzi, , M. Morini, and C. Saletti, “Predictive controller for refrigeration system aimed to electrical load shifting and energy storage,” *Energies*, vol. 15, no. 19, 7125, 2022.
[13] Y. Zhao, Y. Li. S. Dehghan, and Y. Chen, “Learning feedforward control of a one-stage refrigeration system,” *IEEE Access*, vol. 7, pp. 64120-64126, 2019.
[14] A. R. Moreno, “PID control as a process of active inference applied to a refrigeration system, M.S., Dept. Electron. Syst., Aalborg Univ., Copenhagen, 2021.
[15] R. M. Kagami, G. Reynoso-Meza, E. A. P. Santos, and R. Z. Freire, “Control of refrigeration system benchmark problem: an approach based on COR metaheuristic algorithm and TOPSIS method,” *IFAC-PapersOnline*, vol. 52, no. 11, pp. 85-90, 2019.
[16] S. Q. Ain, M. Saifizi, S. M. Othman, A. A. Aziz, W. A. Mustafa, and W. Khairunizam, “OS32-6 temperature control using fuzzy controller for variable speed vapor compression refrigerator system,” vol. 27, pp. 996-1002, 2022.
[17] J. M. Lemos, L. M. Rato, and N. da Silva, “Experience of a predictive adaptive controller on pilot and industrial plants with transport phenomena,” *Advanced Control of Processes with Transport Phenomena*, 1-8.
[18] A. Fattah, “Design of and analysis of speed control using hybrid PID-Fuzzy controller for induction motors,” M.S. thesis, Western Michigan University, Kalamazoo, USA, 2015.
[19] P. C. Eze, B. O. Ekengwu, N. C. Asiegbu, and T. I. Ozue, “Adjustable gain enhanced fuzzy logic controller for optimal wheel slip ratio tracking in hard braking control system,” *Advances in Electrical and Electronic Engineering*, vol. 19, no. 3, pp. 231 – 242, 2021.
[20] B. C. Agwah and P. C. Eze, “An intelligent controller augmented with variable zero lag compensation for antilock braking system,” *International Journal of Mechanical and Mechatronic Engineering*, vol. 16, no. 11, pp. 303 – 310, 2022.
[21] S. O. Banjo, B. O. Bolaji, O. A. Oyelaran, P. O. Babalola, A. S. Afolau, E. Y. Salawu, and M. E. Emetere, “Peformance assessment of a refrigeration system with an integrated condenser under different environment conditions,” *Heliyon*, vol. 10, e29226, 2024.
[22] G. D. Librado, E. E. R. Vázquez, L. A. M. Santiyanes, J. H. P. Vázquez, and C. A. N. Martin, “Transient analysis of a single-stage vapor compression refrigeration system using lumped parameter approaches,” *International Journal on Advances in Systems and Measurements*, vol. 11, no 3 & 4, 2018
[23] W. Bolton, Mechatronics, electronic control in the mechanical and electrical engineering systems, 5th ed., Alfaomega: Mexico D. F., pp. 261-264, May 2013.
[24] B. O. Ekengwu, P. C. Eze, C. N. Asiegbu, C. O. Olisa, and C. F. Udechukwu, Satellite dish antenna control for distributed mobile telemedicine nodes, *International Journal of Informatics and Communication Technology*, vol. 11, no. 3, pp. 206-217, 2022,
[25] L. Frank, L. D. Vrabie, and V. L. Syrmos, *Optimal Control*. NJ: John Wey and Sons, 2012.
[26] L. A. Aloo, P. K. Kihato, and S. I. Kamau, DC servomotor-based antenna positioning control system design using hybrid PID-LQR controller, *European International Journal of Science and Technology*, 5, 17–31, 2016.

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