

Case Study of Various Parameters by Applying Swing up Control for Inverted Pendulum

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Abstract

This paper investigates behavior of the system in terms of time response e.g., steady state error, rise time & overshoot & then compare it with FLC. Being an unstable system, Inverted Pendulum is very common control problem being assigned to control its dynamics. It is almost impossible to balance a pendulum in the inverted position except applying some force from outside to the system.

Keywords: inverted pendulum, swing up control, nonlinear control, PID controller, FLC controller

1. Introduction

In this problem, the pendulum is first placed in upright position, i.e., in a position of unstable equilibrium, or it is given some initial displacement. The controller is then switched in to balance the pendulum and to maintain this balance in the presence of disturbances. A normal disturbance may be a tap on the balanced pendulum. An inverted pendulum is a classic control problem. The process is non linear and not stable with single input signal and many output signals. Our aim is to balance the pendulum vertically on a wagon which is run by motor. The following figure shows an inverted pendulum. The target is to move the wagon along the x direction to a required point without letting the pendulum fall. The wagon, which is run by a DC motor, is controlled by a controller (in our implementation which is analog in nature). The x position of the wagon and the pendulum angle θ are measured and fed to the control system. A force which creates disturbance can be applied on top of the pendulum.

2. Mechanical Set Up for Physical System

An inverted pendulum based problems are renowned as the classical problems in control systems and dynamics and being hugely used for testing control philosophy of the PID controllers and SFB etc. Balance of the motor driven cart and pendulum has internal relation to rocket science and missile technology where the centre of gravity lies beneath the centre of drag which leads to aerodynamic instability. For a real example of control system, our current focus must be on the analysis and development of an inverted pendulum on a cart which is driven by motors. A diagram is given in the Figure 1.

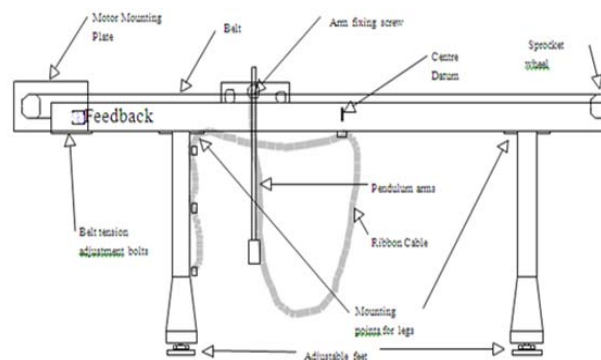


Figure 1. The Pendulum System

For studying the whole model in a proper way, the mathematical model is very much helpful. In this paper the mathematical model of the system can be found with the help of Euler-Lagrange's equation. The resultant non-linear model is made linear then, after that the cart combining with an inverted pendulum, given below is done with an impulse force F . The dynamic equation of motion is made linear with the pendulum angle θ . The physical data [9] of the system are given in Table 1.

Table1. Parameters of the system from feedback instrument .U.K.

Parameter	Value	Unit
Cart mass(M)	0.815	Kilo gram
Mass of the pendulum(m)	0.210	Kilo gram
Length of pendulum(L)	0.305	Meter
Coefficient of frictional force(B)	0.005	Ns/m
Pendulum damping coefficient(D)	0.005	Mm/ radian
Moment of inertia of pendulum(I)	0.099	Kg/m ²
Gravitation force(G)	9.8	m/s ²

3. Mathematical Equation of the System

The Lagrangian equation of the entire system is given as:

$$L = \frac{1}{2}(m\dot{x}^2 + 2ml\dot{x}\dot{\theta}\cos\theta + ml^2\dot{\theta}^2 + M\dot{x}^2) + \frac{1}{2}l\dot{\theta}^2 - mgl\cos\theta$$

The Euler-Lagrange's equation for the cart & resultant system is given as:

$$\begin{aligned} \frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right) - \frac{\delta L}{\delta x} + b\dot{x} &= F \\ \frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}}\right) - \frac{\delta L}{\delta \theta} + d\dot{\theta} &= 0 \end{aligned}$$

Using these two above equations and putting the system parameters value we get:

$$\begin{aligned} (I + ml^2)\ddot{\theta} + ml\cos\theta\ddot{x} - mgl\sin\theta + d\dot{\theta} &= 0 \\ (M + m)\ddot{x} + ml\cos\theta\ddot{\theta} - ml\sin\theta\dot{\theta}^2 + b\dot{x} &= F \end{aligned}$$

The above equation shows the dynamics of the system.

4. Linearization of the Equation

When pendulum is in upright position $\sin\theta = \theta$, $\cos\theta = 1$, $\dot{\theta}^2 = 0$ Using above relation we can write as, To obtain the transfer function of the linear system equations analytically, we must first take the Laplace transform of the system equations. The Laplace transforms are:

$$\begin{aligned} (M + m)X(s)s^2 + qX(s)s + q\theta(s)s^2 &= F(s) \\ (I + ml^2)\theta(s)s^2 - k\theta(s) + qX(s)s^2 &= 0 \end{aligned}$$

Now it becomes:

$$\begin{aligned} r\ddot{\theta} + q\ddot{x} - k\theta + d\dot{\theta} &= 0 \\ p\ddot{x} + q\ddot{\theta} + b\dot{x} &= F \end{aligned}$$

Where, $(M+m) = p$, $mgl = k$, $ml = q$, $I + ml^2 = r$

5. Transfer Function Modelling

After taking Laplace transform of linear differential equation we get the following T.F. model:

$$\frac{\theta(s)}{F(s)} = \frac{-mls^2}{(I+ml^2)s^2 - mgl + ds} \quad [\text{Angle T.F.}] \quad (1)$$

So Equation (1) may be rearrange as:

$$\begin{aligned} &= \frac{-mls^2}{[(M+m)(I+ml^2)-(ml)^2]s^4 + b(I+ml^2)s^3 - mgl(M+m)s^2 - bmgls} \\ &= \frac{-mls^2}{0.06405s^2} \\ &\quad \& \\ \frac{X(s)}{F(s)} &= \frac{rs^2 - k + ds}{(pr - q^2)s^4 + (pd + br)s^3 + (bd - pk)s^2 - kbs} \quad [\text{Cart T.F.}] \quad (2) \\ \frac{X(s)}{F(s)} &= \frac{0.1185s^2 - 0.6276 + 0.005s}{0.12559s^4 + 0.005717s^3 - 0.643357s^2 - 0.003138s} \end{aligned}$$

6. MATLAB Simulation

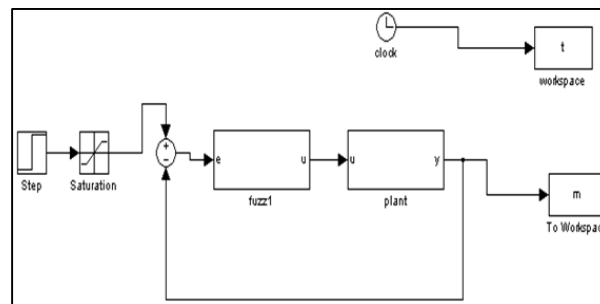


Figure 2. Simulink Diagram of PID & FLC Controller

7. Simulation Result

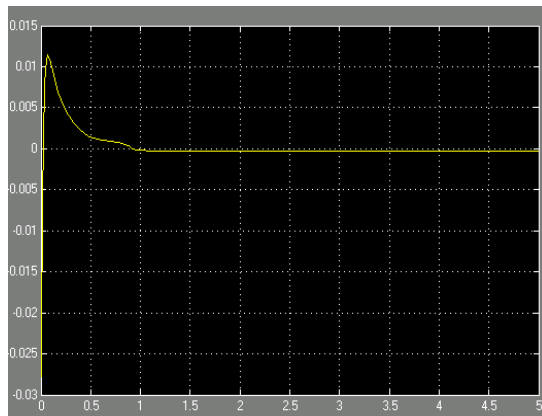


Figure 3. Response For FLC Controller

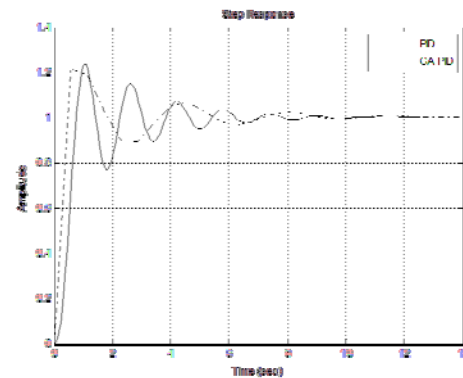


Figure 4. Response of PID Controller For angle

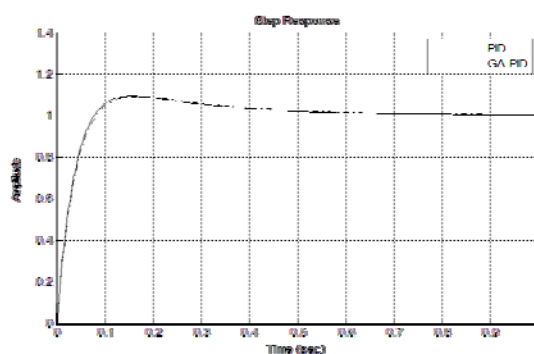


Figure 5. Response of PID Controller For cart

8. Comparison of Various Parameters

Table 2 indicates the comparison between various parameters of PID & FLC Controller.

Table 2. Comparison of Various Parameters

Parameter	FLC	PID
Overshoot	Less	More
Rise Time	More	Less
Settling Time	Less	More
Transient	Not Present	Present

9. Conclusion

Simulation of inverted pendulum using different controller set up shows that system is unstable with non-minimum phase zero. Unlike the conventional PID controller the Fuzzy Logic Controller has some benefits on the system response. It has been seen that FLC using a few number of rules and straightforward implementation used to solve a classical control problem with unknown dynamics. As a future work one can develop design a FLC Controller for double beam inverted pendulum.

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References

- [1] HJT Smith. *Experimental study on inverted pendulum in April 1992. IEEE*. 1991.
- [2] Alan Bradshaw, Jindi Shao. Swing-up control of inverted pendulum systems. *Robotica*. 1996; 14: 397-405.
- [3] Elmer P Dadios. *Fuzzy Logic – Controls, Concepts, Theories and Applications*. First Edition. Janeza Trdine 9, 51000 Rijeka, Croatia. 2012: 428.
- [4] Mario E Magana, Frank Holzapfel. Fuzzy –Logic Control of an inverted pendulum with Vision Feedback. *IEEE transactions on education*. 1998; 41(2): 1998.
- [5] KJ Astrom, K Furuta. Swinging up a pendulum by energy control. *Automatic* 36. 2000: 287-295.
- [6] Feedback instrument. U.K.
- [7] IJ Nagrath, M Gopal. *Control Systems Engineering*. Fourth edition. 1975.
- [8] Ogata. *Modern Control Engineering*. Fourth Edition. 2006.
- [9] Kyung-Jae Ha, Hak-Man Kim. A Genetic Approach to the Attitude Control of an inverted pendulum system. *IEEE*. 1997.
- [10] Felix Grasser, Aldo D'Arrigo, Silvio Colombi, Alfred C Rufer. JOE: A Mobile, Inverted Pendulum. *IEEE transactions on industrial electronics*. 2002; 49(1).

- [11] Zdenko Kovačci, Stjepan Bogdan. Fuzzy Controller Design Theory and Applications. CRC Press Taylor & Francis Group. 2006: 392.
- [12] User's Guide of Matlab for Fuzzy Logic Toolbox. 2012.
- [13] SN Sivanandam, S Sumathi, SN Deepa. Introduction to Fuzzy Logic using MATLAB. Springer-Verlag Berlin Heidelberg. 2007: 441.
- [14] Laxmidhar Bhera, Indrani Kar. Intelligent Systems and Control. 2nd edition. 2010.
- [15] Co Tomas B. Ziegler Nichols Method. Michigan Technological University Department of Chemical Engineering Website. URL: <http://www.chem.mtu.edu/~tbco/cm416/zn.html> (cited February 3, 2010).