Indonesian Journal of Electrical Engineering and Informatics (IJEEI)
Vol.1, No.2, June 2013, pp. 43~48
ISSN: 2089-3272

Problems of Choquet Integral Practical Applications

Sergey Sakulin*[1], Alexander Alfimtsev[2]
[1,2]Bauman Moscow State Technical University,
Russian Federation, 105005, Moscow, 2 Baumanskaya st., 5-1, telp/fax +7-499-267-65-37
Corresponding author, e-mail: sakulin@bmstu.ru

Abstract

Choquet integral with respect to fuzzy measure is a generalization of weighted arithmetic mean aggregation operator. It allows taking into account the phenomenon of dependence between criteria. Due to this it is possible to reflect the expert knowledge more accurately without making the model simplification which is the assumption of independence of the aggregation criteria. The problems of Choquet fuzzy integral applications and possible ways of overcoming them are discussed. Practical applications for this relatively new apparatus are reviewed.

Keywords: *fuzzy logic, fuzzy measure, choquet integral*

1. Introduction

Fuzzy measures and integrals were proposed in the monograph «Theory of capacities» [1] published by Gustave Choquet in 1953 which happened earlier than publication of well-known article [2] by Lotfi Zadeh, the founder of the theory of fuzzy sets took place. In this paper Choquet proposed the use of non-additive measures which he called the capacities. Although there is no direct connection between the theory of fuzzy measures and fuzzy sets theory, they are well combined in sense that the fuzzy integral is a convenient tool to aggregate the values of membership functions of fuzzy sets. Later Choquet's ideas were developed by Sugeno in his unpublished thesis [3] referred in many later works. Sugeno proposed two types of aggregation operators based on Choquet measures. One of these types is called fuzzy discrete Choquet integral and the second is called fuzzy discrete Sugeno integral. As it is said later in this article, the words "fuzzy" and "discrete" are often omitted for brevity. Sugeno integral is used to aggregation for which the result depends on criteria values order on the real axis (ordinal scale aggregation) [4]. Result of aggregation using Choquet integral depends on the value of each criterion [4, 5].

2. Aggregation, Fuzzy Measures and the Choquet Integral

According to [4, 5] numeric criteria aggregation is a method combining them into a single numeric criterion (aggregation result) for the expression of the cumulative effects of these criteria. Aggregation is used in fuzzy inference, pattern recognition, and multi-criteria decision-making problems. Aggregation operator is often called a function of variables (criteria) having some desired properties, each of variables being defined in the interval [0,1]. The domain of this function is also the interval [0,1]. Fuzzy measure expresses the subjective weight or importance of each subset of criteria and defined as follows [4].

Fuzzy (discrete) measure is a function $\psi: 2^J \rightarrow [0,1]$, where 2^J is the set of all subsets of the criteria index set $J = \{1,..., H\}$, which satisfies the following conditions:

1) $\psi(\varnothing) = 0,\ \psi(J) = 1$;

2) $\forall D, B \subseteq J:\ D \subseteq B \Rightarrow \psi(D) \leq \psi(B)$ ■

Further, we will omit the curly brackets writing $i,\ ij$ instead of $\{i\},\ \{i, j\}$ respectively. Instead of the "criterion of the index $i \in J$" we will also use the "criterion i" instead of the "criteria index set J" we will use the "set of criteria J" both done for brevity reason.

Received January 19, 2013; Revised March 23, 2013; Accepted April 11, 2013

Firstly, we consider the basic concepts used in the fuzzy measures theory. Shapley [6] proposed a definition of the criterion importance coefficient based on several natural axioms. In the context of the fuzzy measures theory Shapley index for the criterion $i \in J$ with respect to fuzzy measure ψ is determined by the following expression:

$$\Phi_{Sh}(i) := \sum_{D \subseteq (J-i)} \frac{(|J|-|D|-1)!|D|!}{|J|!} \left[\psi(D \cup i) - \psi(D) \right] \quad (1)$$

Murofushi and Soneda proposed an interaction index between criteria [7]. This index is used to express the sign and degree of interaction between criteria and is determined by the following expression:

$$I(i,j) := \sum_{D \subseteq (J-\{i,j\})} \frac{(|J|-|D|-2)!|D|!}{(|J|-1)!} \left[\psi(D \cup ij) - \psi(D \cup i) - \psi(D \cup j) + \psi(D) \right] \quad (2)$$

3. Formalization of Dependencies between Criteria

Marichall [8] identified the main types of dependencies between criteria in the context of aggregation with Choquet integral.

Correlation is the best known of the dependencies between criteria. Two criteria $i, j \in J$ are positively (negatively) correlated if expert can observe a positive (negative) correlation between the contributions of two criteria to the aggregation result.

Substitutiveness (complementarity) is another type of dependence. The idea of formalizing this type of dependencies using fuzzy measures was proposed by Murofushi and Soneda [7]. Considering again two criteria $i, j \in J$ we can suppose that the expert believes that satisfying only one criterion causes almost the same effect as satisfying of both. Here the importance of a pair of criteria is close to the importance of each of them individually, even if other criteria are present. In this case we see that criteria i and j almost substitutive or interchangeable.

Preferred dependence (preferred independence) is the type of dependency which is well known in the multiattribute utility theory [9, 10]. We suppose that expert's preferences on the set of criteria realizations A are known and expressed as the partial weak order over A. The set A is usually consists of parameters available for assessment objects. Denote $\mathbf{g}_D$ the realization of criteria g_i where $i \in D$, denote $\mathbf{g}_{J-D}$ the realization of criteria g_i where $i \in J - D$. The subset $D \subset J$ of criteria is said to be preferentially independent of $J - D$ if for all pair $\mathbf{g}_D$, $\mathbf{g}'_D$ we have from $(\mathbf{g}_D, \mathbf{g}_{J-D}) \succeq (\mathbf{g}'_D, \mathbf{g}_{J-D})$ for some realization $\mathbf{g}_{J-D}$ follows $(\mathbf{g}_D, \mathbf{g}_{J-D}) \succeq (\mathbf{g}'_D, \mathbf{g}_{J-D})$ for all realizations $\mathbf{g}_{J-D}$. Otherwise subset $D \subset J$ preferably depends on the subset $J - D$. The full set of criteria J mutually preferably independent if subset D preferably independent of subset $J - D$ for each subset D. It is known [4, 9, 10] that if certain criteria are preferably dependent on others then the additive aggregation operators can not reflect the expert's preferences. In particular, in this case it is impossible to use the weighted arithmetic mean operator.

4. Problems in Practical Applications and Possible Ways to Overcome them

According to Grabisch [11] "From the beginning of the application of fuzzy measures and integrals to multicriteria evaluating problems, it has always been felt that the non-additivity of fuzzy measure was able to model dependency between criteria, but until recently, this point was not investigated in a rigorous manner, for nobody defined precisely what he intends by "dependent". If a fuzzy measure is additive, the criteria do not interact with each other and the interaction indices (2) of these criteria are equal to zero. Therefore, if the expert thinks the criteria mutually preferably independent, the corresponding interaction indices are equal to zero.

If the expert suggests that the criteria are preferably dependent then it is possible to formalize this only by means of partial weak order on the set of criteria realizations (training set). No other method of formalization of the criteria preferred dependence and independence has not been proposed.

To use the Choquet integral preliminary we have to identify the fuzzy measure on the basis of expert knowledge. This identification is complicated by exponential increasing complexity in the sense that it is necessary to set a value of fuzzy measure for each subset of criteria. Setting the values of all 2^H coefficients of the fuzzy measure $\psi(D),\ D \subseteq J$ is very difficult or even impossible for the expert. Note that even in case of three criteria for determining the fuzzy measure it is necessary to obtain $2^3 = 8$ coefficients. Despite this complexity Choquet integral still can be applied in practice. For this Grabisch proposed the concept of κ-order fuzzy measure or κ- additive fuzzy measure [12]. This order κ can be less than the number of aggregated criteria, $\kappa < |J| = H$. Essence of the κ- additivity concept consists in simplification of the task of fuzzy measures determining by excluding from consideration the dependencies between more than κ criteria. According to the κ- additivity concept in most practical cases it is possible to use the Choquet integral with respect to 2-order fuzzy measure or, equivalently, the 2-order Choquet integral because it allows to model the interaction between the criteria while remaining relatively simple [12]. The paper [13] is entirely devoted to the question under what conditions such a simplification (using of the 2-order Choquet integral) is correct. This paper presents necessary conditions that should satisfy the expert preferences in order that they can be formalized using the 2-order Choquet integral.

In addition to increasing complexity there also appears a problem of the expert's understanding of fuzzy measure coefficients meaning [11]. To solve this problem Grabisch [14] proposed the idea of the graphical interpretation of the 2-order Choquet integral. This interpretation is represented by a constraint line of values of the interaction index and Shapley indexes for two criteria on the coordinate plane. This idea has been applied to identify the fuzzy measure using a hierarchical diagram of pairwise comparisons ("diamond pairwise comparisons method") [15, 16]. This approach to fuzzy measure identification has two main difficulties. First, because the expert considers each pair of criteria separately he (she) does not have a complete picture of aggregation and can formulate his (her) preferences so that the fuzzy measure identification problem based on these preferences obviously will not have the solution. Second, the "diamond" form of scale is not trivial for the expert. These difficulties can be overcome by an apparent indication of the restrictions imposed on expert preferences [13] and also by careful tutoring of the expert of a graphic interpretation method.

Visualization of 2-order Choquet integral offered in [17] can be both alternative and addition of graphic interpretation. This visualization is based on one-to-one comparison between mathematical object (2-order Choquet integral) and physical object (the lever fixed in the center by a spring with a simple stiffness factor which can rotate round a horizontal axis). Loads with weights corresponding to interaction indices $I(ij)$ (2) and fuzzy measures $\psi(i)$ are established on the lever. This approach relies on the natural intuition peculiar to many people in regard to the well-known physical object and allows the expert to have a clear intuitive understanding of behavior of 2-order Choquet integral. This visualization reveals the expert preferences in the form of limitations on the fuzzy measure. Fuzzy measure can be identified based on these limitations and implemented in order to build a lever. This process is iterative and continues until 2-order Choquet integral with respect to identified fuzzy measure will satisfy the expert. This approach is also facing difficulties. First, when the number of criteria (from about four to five with additional loads that are associated with interaction indices) the expert is having difficulty accepting this visualization (as we know from psychology the average person is able to simultaneously keep in attention no more than 7 items). Second, such visualization considers fuzzy measure of each separate criterion and Shapley indices (1) are visualized for each criterion separately. The above puts extra pressure on the experts' attention. It seems that both of these difficulties can be overcome by careful design of procedures for working with the expert taking into account specifics of each subject domain.

In the process of expert knowledge formalization using different approaches we need to select a mathematical method for the fuzzy measure identification. These methods differ in the types of information that is required as input. Review of methods for fuzzy measure identification in relation to the utility theory is presented in [18].

Method based on least squares is not well suited for solving practical problems because it requires desired values of aggregation result as input. But the experts are not always able to set such values.

Method based on the maximum split is well suited to meet the challenges of recognition, as it maximizes the minimum difference between the results of the aggregation of the training set. The expert describes an sample of each class and ranks them by non-strict order that serves as input to the identification method [19].

Method based on minimization of fuzzy measure variance or maximization of fuzzy measure entropy is the most suited for solving many practical problems [20]. This method is based on the principle of maximum entropy proposed in 1957 by Jaynes [21]. In relation to the construction of aggregation operators that principle involves the use of all available information about the aggregation criteria but the most unbiased attitude to the inaccessible information. Kojadinovic [20] extended the principle of maximum entropy on the utility theory and developed fuzzy measures identification method based on this.

Fuzzy measure identification by one or another method requires setting indifference threshold values for interaction index and Shapley index and the aggregation result. Usually, this question receives little attention, and it is considered that the expert must specify these values on the grounds of the necessary accuracy [18]. But in practice, the indifference threshold values can be set in such a way that it will cause absence of the fuzzy measure which in its turn is the solution of the identification problem. Way to prevent such situations proposed in [17].

5. Review of Current Applications

The following practical examples briefly describe the application of fuzzy measures and the Choquet integral including interface properties evaluation, technical diagnostics, navigation, and image processing.

Authors of the article [22] proposed a solution to the problem of determining the degree of software interface usability with the help of this aggregation operator. This assumes direct expert determination of fuzzy measures by filling in special tables for multiple criteria (about four). For the expert, this method is very difficult task in the case of even a minor increase in the number of criteria. However, this example shows that the use of Choquet integral can improve the accuracy of interface usability evaluation.

Another practical example of the Choquet integral and fuzzy measure application is the analysis of the technological processes state based on fuzzy expert knowledge [17]. The first level of state analysis evaluates the membership functions values of the process parameters. These fuzzy sets are based on the expert knowledge of the process faults. At the second level we obtain membership values of the current process state to a particular class of fuzzy states by aggregating membership functions values using Choquet integral and the fuzzy measures which is, for example, a class of equipment proper functioning states. Fuzzy measure identification is realized by the maximization fuzzy measure entropy method using visualization. This example confirms the possibility of increasing the accuracy of classification technological processes state to the class of equipment proper functioning states or classes corresponding to the process faults.

Yet another example of application of this mathematical tool is illustrated the article [23] which describes the navigation system for pedestrians. The inputs to the system are the subjective assessment of various characteristics of the routes in particular: distance, quality of the road surface, neighborhood picturesque, degree of noise etc. All of these criteria are often linked in a nontrivial way. Therefore, the aggregation of such criteria is conveniently carried out by using the Choquet integral. As a method for the fuzzy measure identification there the least squares method is applied. Despite relatively high complexity of implementation, this example shows the flexibility of the Choquet integral as an aggregation operator of such subjective criteria.

Choquet integral with respect to fuzzy measure is also used in the field of image processing. Article [24] describes recognition of areas of interest on 3D-tomographic images of electrotechnical parts made of composite materials. As fuzzy measure identification method the relative entropy method is used. Relative entropy method is the development of a variance minimizing method. It adapted for the purposes of recognition and allows getting better results. Four attributes derived from tomographic images are aggregated. The experimental results confirm perspectivity of use related apparatus in the image recognition.

Another area of application of Choquet integral is improvement of digital images. Authors of the article [25] proposed a method for digital image improving based on the use of the Choquet integral. The experimental results showed that this method can process images with high accuracy comparable to the popular filtration methods, besides the greatest accuracy of processing was obtained using the fuzzy measure identification method based on the maximization fuzzy measure entropy.

6. Conclusion

The paper considers the practical applications of fuzzy measure and the Choquet integral and analyzes problems in these applications. The main hindrance on a way of wide practical use of these tools is related to the problems to occur during the work with the expert on formalization of his knowledge in the form of fuzzy measure coefficients. It seems possible to overcome these difficulties by using methods of visualization adapted for each separate area of practical applications. Currently, a field of research related to fuzzy measures and integrals is developing intensively.

References

[1] Choquet G. Theory of capacities. *Annales de l'Institut Fourier*. 1953; 5: 131-295.
[2] Zadeh L, Fuzzy sets. *Information and Control*. 1965; 8: 338–353.
[3] Sugeno M. Theory of fuzzy integrals and its applications. PhD Thesis. Tokyo: 1974.
[4] Grabisch M, Orlovski S, Yager R. Fuzzy aggregation of numerical preferences. In: R. Slowinski. *Editor*. Handbook of Fuzzy Sets Series. Dordrecht: Kluwer Academic; 1998: 31-68.
[5] Detyniecki M. Mathematical Aggregation Operators and their Application to Video Querying. PhD Thesis. Paris: 2000.
[6] Shapley L. A value for n-person games In: Kuhn H, Tucker A. *Editors*. Contributions to the Theory of Games. Princeton: Princeton University Press; 1953: 307–317.
[7] Murofushi T, Soneda S. *Techniques for reading fuzzy measures (III): interaction index.* 9th Fuzzy System Symposium. Sapporo. 1993; 693–696.
[8] Marichal JL. An axiomatic approach to the discrete Choquet integral as a tool to aggregate interacting criteria. *IEEE Transactions on Fuzzy Systems*. 2000; 8(6): 800-807.
[9] Moulin H. Axioms of cooperative decision making. Cambridge: Cambridge university press. 1988: 480.
[10] Fishburn P. Utility theory for decision making. New York: John Willey & Sons, Inc. 1970: 337.
[11] Grabisch M. The application of fuzzy integrals in multicriteria decision making. *European Journal of Operation Research.* 1996; 89: 445-456.
[12] Grabisch M. k-order additive discrete fuzzy measures and their representation. *Fuzzy Sets & Systems*. 1997; 92: 167–189.
[13] Mayag B, Grabisch M, Labreuche Ch. A representation of preferences by the Choquet integral with respect to a 2-additive capacity. *Theory and Decision.* 2011; 71: 297-324.
[14] Grabisch M. A Graphical Interpretation of the Choquet Integral. *IEEE Transactions on Fuzzy Systems.* 2000; 8: 627-631.
[15] Takahagi E. *A fuzzy measure identification method by diamond pairwise comparisons: AHP scales and Grabish's graphical interpretation*. 11th international conference, KES 2007 and XVII Italian workshop on neural networks conference on Knowledge-based intelligent information and engineering systems. Roma. 2007; 316-324.
[16] Wu J, Zhang Q. 2-order additive fuzzy measure identification method based on diamond pairwise comparison and maximum entropy principle. *Fuzzy Optimization and Decision Making.* 2010; 9: 435-453.
[17] Sakulin S, Devyatkov V. Analysis of the technological processes state based on fuzzy expert knowledge (In Russian). Saarbrucken: Lambert Academic Publishing. 2012: 240.
[18] Grabisch M, Kojadinovic I, Meyer P. A review of methods for capacity identification in Choquet integral based multi-attribute utility theory: Applications of the Kappalab R package. *European Journal of Operational Research.* 2008; 186(2): 766-785.
[19] Marichal J, Roubens M. Determination of weights of interacting criteria from a reference set. *European Journal of Operational Research*. 2000; 124: 641-650.
[20] Kojadinovic I. Minimum variance capacity identification. *European Journal of Operational Research*. 2007; 177(1): 498-514.
[21] Jaynes E. Information theory and statistical mechanics. *Phys. Rev.* 1957; 106: 620-630.

[22] Sicilia M, Garsia E, Calvo T. An Inquiry-Based Method for Choquet Integral-Based Aggregation of Interface Usability Parameters. *República Checa Kybernetica*. 2003; 39(5): 601-614.
[23] Akasaka Y, Onisawa T. Pedestrian Navigation Reflecting Individual Preference for Route Selection - Evaluation on Fitness of Individual Preference Model. *Journal of Japan Society for Fuzzy Theory and Intelligent Informatics*. 2006; 18(6): 900-910.
[24] Jullien S, Mauris G, Valet L, Teyssier S. *Identification of Choquet integral's parameters based on relative entropy and applied to classification of tomographic images.* IPMU'08 Torremolinos (Malaga). 2008; 1360-1367.
[25] Alfimtsev A, Sakulin S, Devyatkov V. Digital image improvement with use of the Choquet fuzzy operator. *BMSTU Transactions.* 2011; 1: 5-12.