

# Parameter estimation and optimal control strategies for sustainable management of mangrove forests

Nurul Eliana Atika Mohd Idrus<sup>1</sup>, Sie Long Kek<sup>2</sup>

<sup>1,2</sup>Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology,  
Universiti Tun Hussein Onn Malaysia, Pagoh, Johor, Malaysia

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## ABSTRACT

This paper presents an integrated computational framework combining parameter estimation and optimal control for sustainable mangrove forest management. We begin by addressing uncertainty in model parameters using a data-driven solution. A least-squares optimization problem is solved using the Levenberg-Marquardt algorithm to recursively estimate parameters in the dynamic mangrove ecosystem model. Later, we formulate an optimal control problem to drive mangrove populations toward ecological steady states as rapidly and efficiently as possible. To this end, we define the Hamiltonian function and derive the necessary optimality conditions to design optimal control policies. Our key contribution lies in unifying parameter estimation and control synthesis into a single, implementable algorithm tailored for mangrove dynamics. Numerical simulations demonstrate the algorithm's efficacy: parameter estimates achieve high accuracy, as evidenced by low mean squared error, while the optimal control policy minimizes a defined performance index, ensuring ecological and operational efficiency. This integrated approach offers decision-makers a data-driven, mathematically rigorous tool to guide restoration and conservation efforts. By bridging estimation and control, our methodology enhances adaptive management of mangrove forests, supporting long-term ecological resilience and sustainability in dynamic coastal environments.

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## Corresponding Author:

Sie Long Kek

Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology,

Universiti Tun Hussein Onn Malaysia, 84600 Panchor, Johor, Malaysia

Email: slkek@uthm.edu.my

## 1. INTRODUCTION

Mangrove forests are among the most productive and ecologically significant coastal ecosystems, providing critical services such as carbon sequestration, coastal protection, biodiversity support, and livelihoods for millions of people worldwide [1, 2]. Despite their immense value, mangroves are under severe threat from anthropogenic activities—including aquaculture expansion, urban development, and pollution—as well as climate-induced stressors like sea-level rise and altered precipitation patterns [3, 4]. The accelerating rate of mangrove loss not only undermines ecosystem resilience but also jeopardizes the socioeconomic stability of coastal communities that depend on these habitats.

Effective conservation and restoration of mangrove ecosystems require robust scientific frameworks that integrate ecological dynamics with human decision-making. Mathematical modelling has emerged as a powerful tool for understanding the complex interactions within mangrove systems and for evaluating the potential outcomes of different management interventions [5, 6]. Central to these models are accurate parameter

estimates that reflect real-world ecological processes—such as growth rates, mortality, recruitment, and hydrological influences—which are often uncertain or context-specific. Reliable parameter estimation is therefore a prerequisite for credible predictions and informed policy design.

In previous studies, Singh and Mahajan [8] proposed a workflow for producing a mangrove ecosystem map and estimating mangrove above-ground biomass (AGB). They used synthetic aperture radar data and a machine learning algorithm to estimate the mangrove AGB level in the village of Guneri's mangrove forest. Zheng and Takeuchi [9] presented a remote sensing (RS)-based productivity model to estimate the light use efficiency (LUE) and gross primary production (GPP) of mangrove forests in China. The results demonstrated the potential of the RS-based productivity model for scaling up GPP in mangrove forests, a key to exploring the carbon cycle of mangrove ecosystems at national and global scales. Kejeh et al. [10] used a computational approach to model the impact of artisanal hydrocarbon on the biodiversity of a mangrove forest. The numerically simulated database showed a monotonic decrease in relative abundance after 10 months in the experimental study.

In parallel, optimal control theory offers a systematic approach to identifying management strategies that balance ecological sustainability with socioeconomic objectives over time [11]. By formulating objective functions that incorporate the costs of intervention and the benefits of conservation, optimal control models can guide decisions about when and how intensively to implement actions such as reforestation, pollution control, or protected area enforcement. When coupled with dynamic models of mangrove ecosystems, these strategies can be tailored to local conditions and adaptive to changing environmental contexts, thereby enhancing long-term resilience.

Hasan et al. [11] employed a control theory model to address the consequences of toxicity and illegal logging of mature trees in the Sundarban ecosystem, the largest mangrove forest in the world. They reflected on the modified Leslie-Gower response function to establish it as an alternative resource for industries when forestry resources were devastated. Paul et al. [12] used a three-compartment mathematical model to study the dynamics of greenhouse gas emissions, concentrations, and uptake. They installed a chemical reactor system near the power plant and naturally afforesting the regions. They found that the chemical reactor system, as the control strategy, is the best way to minimise air pollution. Sadi et al. [13] proposed a novel hierarchical cascaded model predictive control (MPC) system for altitude, attitude and battery efficiency for a quadrotor in a mangrove area. The control system addresses computational complexity by decomposing the overall MPC strategy into two different schemes to enhance the unmanned aerial vehicle's resilience to wind turbulence, a significant disturbance factor in mangrove environments.

In addition, extended Kalman filtering (EKF) and moving horizon estimation (MHE) are widely used in biological and ecological systems for parameter estimation, tracking nonlinear ecosystem states, and managing uncertainties in environmental variables. Zhang et al. [14] addressed the limitations of current discharge estimation methods by integrating the EKF with a hydrological model. They proposed a novel data assimilation framework to enhance the accuracy of river discharge estimation. Hinkel and Pourkargar [15] developed an optimization-based nonlinear output-feedback control framework for semi-closed greenhouse systems. They integrated MHE and MPC to optimize crop growth and operating costs by regulating greenhouse environmental conditions to achieve desired steady-state conditions. Karimanzira and Rauschenbach [16] combined an MHE and an unscented Kalman filter (UKF) for state estimation in flood forecasting. They investigated a conceptual rainfall-runoff model for streamflow prediction of watersheds with a fast response to heavy rain. The results showed that the UKF and the MHE produced almost similar results, with UKF slightly better.

This paper addresses the dual challenges of parameter uncertainty and strategic decision control in mangrove management. We propose an efficient computational algorithm for parameter estimation and optimal control in mangrove management. We consider a dynamic model of a mangrove forest, assuming that the model parameters are unknown. Then, we introduce a least-squares optimisation problem, and apply the Levenberg-Marquardt method to estimate the model parameters. After that, we form an optimal control problem with the parameter estimates to derive cost-effective management policies that promote sustainable mangrove conservation. By combining parameter estimation and optimal control, our approach provides a practical implementation in the mathematical modelling of mangrove forest management. We demonstrate an example of mangrove forest dynamics using the proposed computational algorithm. The simulation results showed the accuracy of model parameter estimates and the efficiency of control strategies. Therefore, by bridging ecological modelling and decision science, our study supports evidence-based policies that safeguard mangrove ecosystems for future generations.

The paper is organised as follows: Section 2 describes the problem of mangrove forest management and presents the mangrove forest dynamic model. Then, the parameter estimation and optimal control techniques are discussed for handling the model. Section 3 illustrates an example of model management and demonstrates the computational algorithm's practicality by combining parameter estimation and optimal control in mangrove forest management. Finally, concluding remarks are made on the sustainable management of mangrove forests.

## 2. METHOD

Consider a dynamic model for mangrove forest, which is also known as a simple age-structured forestry biomass model [11], given as follows,

$$\frac{dx_1}{dt} = rx_1 \left(1 - \frac{x_1}{k}\right) - \beta x_1 + \gamma x_1 - \tau_1 x_1^3, \quad (1)$$

$$\frac{dx_2}{dt} = e\beta x_1 - d_1 E x_2 - d_2 x_2 - p x_2 - \tau_2 x_2^2, \quad (2)$$

$$\frac{dx_3}{dt} = \left(\alpha_1 - \frac{\alpha_2 x_2}{\alpha + x_2}\right) x_3 - d_3 x_3, \quad (3)$$

for  $t = 0, \dots, 10$  years, where the state variables are defined below,

- $x_1$  denotes the population of immature trees (Juvenile),
- $x_2$  denotes the population of mature trees (Adult),
- $x_3$  denotes the number of industries (Industrialization).

The model parameters with their ecological description and their respective dimensionless numerical values are shown in Table 1. This dynamic model has concerned the presence of harvesting, toxicity and illegal logging.

Table 1. Description of parameters of mangrove forest model

| Parameter  | Description                                                           | Value |
|------------|-----------------------------------------------------------------------|-------|
| $r$        | Intrinsic growth rate of immature trees                               | 1.5   |
| $k$        | Environmental carrying capacity of forestry biomass                   | 100   |
| $\beta$    | Transition rate from immature to mature                               | 0.9   |
| $\gamma$   | Rate of new plantation                                                | 3.5   |
| $e$        | Rate of conversion                                                    | 0.99  |
| $d_1$      | Depletion rate of the mature tree                                     | 0.12  |
| $d_2$      | Natural depletion rate of a mature tree                               | 0.1   |
| $d_3$      | Decreasing rate of industries in the absence of preferred mature tree | 0.785 |
| $E$        | Harvesting effort                                                     | 3.8   |
| $p$        | Illegal logging of immature trees                                     | 0.1   |
| $1/\alpha$ | Average forestry protection rate                                      | 1/2   |
| $\alpha_1$ | Measures the reduction rate of industrialization                      | 1.5   |
| $\alpha_2$ | Maximum value for the reduced rate of industries can attain           | 3.3   |
| $\tau_1$   | Rate of toxicity for immature trees                                   | 0.4   |
| $\tau_2$   | Rate of toxicity for mature trees                                     | 0.3   |

From the simulation of the dynamic model (1)-(3), the graphical result is shown in Figure 1. The populations of immature trees, mature trees, and industrialization over 10 years, with initial populations of 1.6396, 1.3794, and 0.7322 hectares, is presented. The trajectories of immature trees ( $x_1$ ) and mature trees ( $x_2$ ) are increased in the first two years and then stay along their steady states for the following years. The trajectory of the number of industries ( $x_3$ ) decreases gradually and remains at zero after four years. This trend shows that the tree population is affected by industrial activity.

The presence of industries has affected the ecosystem of the mangrove forest, as industrial activities have produced toxins and reduced the density of the forest population. On the other hand, in the absence of industrialization, the populations of immature and mature trees will remain constant. The steady states for the juvenile, adult, and industry are, respectively, 3.18, 2.17 and  $4.6 \times 10^{-5}$  hectares.

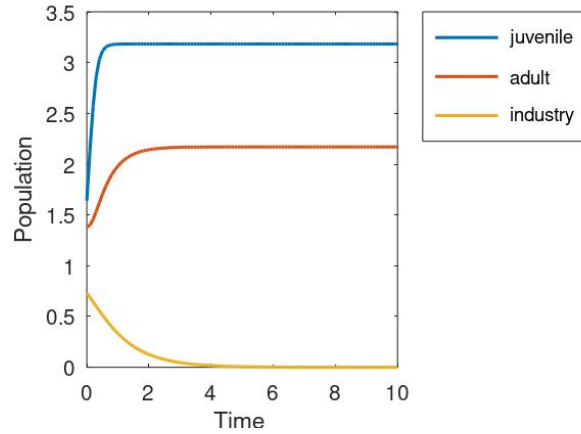


Figure 1. Population of trees and industrialization in mangrove forest model

## 2.1. Parameter estimation

Assume that the values of the model parameters given in Table 1 are unknown, and we only have the data-driven solution of the model (1)-(3) as shown in Figure 1. We introduce a least squares optimization problem [17, 18] as follows,

$$\text{Minimize } J_1(\theta) = \frac{1}{2}(f(x) - f(x, \theta))^T(f(x) - f(x, \theta)), \quad (4)$$

over the dynamic model in (1)-(3), which is formulated by

$$\frac{dx}{dt} = f(x, \theta), \quad (5)$$

where  $f$  represents the dynamic function for (1)-(3),  $x = (x_1, x_2, x_3)^T$  is the state vector,  $\theta$  is the vector of the model parameters mentioned in Table 1, and the term  $^T$  represents the transposition operator. It is assumed that the data-driven solution of  $f(x)$  is known. Thus, the goal of our study is to estimate the value of the model parameter vector  $\theta$  such that the sum of squared errors  $J_1$  is minimized. This problem is referred to as the parameter estimation problem. Notice that the sum of squared errors  $J_1$  in our study is evaluated based on the differences between the data-driven dynamic model solution and the estimated dynamic model solution, which differs from the sum of squared errors in most least squares optimization problems, including nonlinear regression problems. In general, the least squares optimization problems have the sum of squared errors depending on the differences between the real output and the measured output.

Consider the objective function  $J_1$  in the second-order Taylor series expansion [19] about the current parameter  $\theta^i$  at the iteration  $i$ , as follows,

$$J_1(\theta^{i+1}) \approx J_1(\theta^i) + (\theta^{i+1} - \theta^i) \left( \frac{\partial J_1(\theta^i)}{\partial \theta} \right) + \frac{1}{2}(\theta^{i+1} - \theta^i) \left( \frac{\partial^2 J_1(\theta^i)}{\partial \theta^2} \right) (\theta^{i+1} - \theta^i). \quad (6)$$

Calculating the first-order derivative with respect to the next parameter  $\theta^{i+1}$  to (6), we obtain the first-order necessary condition [20, 21],

$$0 \approx \left( \frac{\partial J_1(\theta^i)}{\partial \theta} \right) + \left( \frac{\partial^2 J_1(\theta^i)}{\partial \theta^2} \right) (\theta^{i+1} - \theta^i), \quad (7)$$

and rearrange it to be a normal equation,

$$-\left( \frac{\partial J_1(\theta^i)}{\partial \theta} \right) = \left( \frac{\partial^2 J_1(\theta^i)}{\partial \theta^2} \right) (\theta^{i+1} - \theta^i). \quad (8)$$

Hence, the following recurrence equation

$$\theta^{i+1} = \theta^i - \left( \frac{\partial^2 J_1(\theta^i)}{\partial \theta^2} + \lambda I \right)^{-1} \left( \frac{\partial J_1(\theta^i)}{\partial \theta} \right) \quad (9)$$

is expressed to update the model parameter  $\theta$ , where  $\lambda$  is a damping parameter. The recursion equation (9) is known as the Levenberg-Marquardt equation [22]. Notice that the small value of the damping parameter  $\lambda$  gives the Gauss-Newton updates, which the term  $\lambda I$  can be ignored, while the large value of the damping parameter  $\lambda$  gives the steepest descent updates, which the term  $\lambda I$  becomes an essential diagonal element dominating the inverse matrix. Here, the gradient and Hessian with respect to the model parameter  $\theta$  are given by

$$\frac{\partial J_1(\theta)}{\partial \theta} = - \left( \frac{\partial f}{\partial \theta} \right)^\top (f(x, \theta, t) - f(x, t)), \quad (10)$$

$$\frac{\partial^2 J_1(\theta)}{\partial \theta^2} = - \left( \frac{\partial^2 f}{\partial \theta^2} \right)^\top (f(x, \theta, t) - f(x, t)) + \left( \frac{\partial f}{\partial \theta} \right)^\top \left( \frac{\partial f}{\partial \theta} \right). \quad (11)$$

It is remarked that the first term on the right side of the equal sign in (11) is ignored because its components are negligibly small, so the second term approximates the Hessian.

At the end of iteration, the optimal model parameter, denote it as  $\hat{\theta}$ , is resulted when  $\theta^{i+1} \approx \theta^i$  occurs at the convergence. Thus, the sum of squared error in (4) is minimized, and the model trajectory should track the original trajectory satisfactorily. Moreover, the model parameters are located in the 99% confidence interval, as follows,

$$\hat{\theta} - z_{0.99} \cdot \sigma_\theta \leq \hat{\theta} \leq \hat{\theta} + z_{0.99} \cdot \sigma_\theta, \quad (12)$$

with  $z_{0.99} = 2.326$  and  $\sigma_\theta$  represents the standard deviation of the model parameter  $\theta$ . Hence, the model parameters are assumed to be included in the error bounds given by  $z_{0.99} \cdot \sigma_\theta$ .

## 2.2. Optimal control

Now, we denote the dynamic function (5) as

$$\hat{f}(x, t) = f(x, t | \hat{\theta}), \quad (13)$$

once the model parameter  $\theta$  is estimated, that is  $\hat{\theta}$ . Then, we define an optimal control problem [23, 24] as follows,

$$\text{Minimize } J_2(u) = \frac{1}{2} x^\top(T) S(T) x(T) + \frac{1}{2} \int_0^T (x^\top Q x + u^\top R u) dt \quad (14)$$

subject to

$$\dot{x} = \hat{f}(x, t) + B u \quad (15)$$

with an initial state  $x(0) = x_0$ , where  $u$  is a control variable,  $B$  is the control coefficient matrix and  $J_2$  is the performance index of using the control input, while  $S(T)$ ,  $Q$  and  $R$  are the weighting matrices and  $T$  is the final time. The aim of defining the optimal control problem in (14)-(15) is to find an admissible control  $u$  from a control set  $U$ , that is  $u \in U$ , such that the performance index  $J_2$  is minimized over the nonlinear state equation.

Consider the Hamiltonian function [23],

$$H(x, u, t, p) = \frac{1}{2} (x^\top Q x + u^\top R u) + p^\top (\hat{f}(x, t) + B u), \quad (16)$$

where  $p$  is the multiplier vector to be determined. A set of first-order necessary conditions [23, 25] is derived by

$$0 = H_u(x, u, t, p) = Ru + B^\top p, \quad (17)$$

$$\dot{x} = H_p(x, u, t, p) = \hat{f}(x, t) + Bu, \quad (18)$$

$$-\dot{p} = H_x(x, u, t, p) = Qx + A^\top p, \quad (19)$$

with the boundary conditions  $x(0) = x_0$  and  $p(T) = S(T)x(T)$ , where the state transition matrix  $A$  is linearized from the dynamic function, given by

$$A \approx \nabla_x \hat{f}(x, t), \quad (20)$$

with  $\nabla_x = \partial \hat{f} / \partial x$ . Note that (17) is the stationary condition, (18) is the state equation and (19) is the costate equation. It is remarked that the linearization is required for designing the control law, and the state equation remains in its nonlinear form during the solution procedure.

Using the sweep method [25, 26],

$$p(t) = S(t)x(t), \quad (21)$$

the control function, from (17), can be expressed by

$$u = -Kx, \quad (22)$$

where the feedback gain is

$$K = R^{-1}B^\top S. \quad (23)$$

We form the linearized state equation (18) as

$$\dot{x} = Ax + Bu, \quad (24)$$

and substitute the control (22) into (24) yields

$$\dot{x} = (A - BK)x. \quad (25)$$

Furthermore, from (21), the derivative of the costate variable is given by

$$\dot{p} = \dot{S}x + S\dot{x}, \quad (26)$$

which can be written as

$$\dot{p} = \dot{S}x + S(A - BK)x, \quad (27)$$

after taking into account the state equation (25). Notice that the costate in (27) is equivalent to the costate in (19), which gives the Riccati differential equation

$$-\dot{S} = Q + A^\top S + SA - K^\top RK, \quad (28)$$

with the given boundary value  $S(T)$  when the feedback gain (23) is considered.

Finally, the state equation with the optimal control law (22) is presented below,

$$\dot{x} = \hat{f}(x, t) - BKx, \quad (29)$$

which its solution can be obtained by using the numerical method, such as Euler and Runge-Kutta methods [27, 28]. In addition, the performance index (14) is evaluated by

$$J_2(0) = \frac{1}{2}x^\top(0)S(0)x(0), \quad (30)$$

where  $S(0)$  is the initial value after the Riccati differential equation (28) is solved backwards.

### 2.3. Computational algorithm

From the discussion above, we summarize the calculation procedure as a computational algorithm, which combines parameter estimation and optimal control, to the mathematical modelling of mangrove forest management. The computational algorithm is given as follows.

- Step 1 Solve the dynamic model (1)-(3), and store the model solution as the data-driven solution. Set  $i = 0$  to begin the iteration.
- Step 2 Compute the gradient and Hessian from (10) and (11), respectively.
- Step 3 Update the model parameter from (9), and evaluate the dynamic function from (5). Then, measure the objective function from (4).
- Step 4 Test the convergence. If  $\|\theta^{i+1} - \theta^i\| < \text{tolerance}$ , stop the iteration, and go to Step 5; otherwise, set  $i = i + 1$ , go to Step 2 and repeat the iteration.
- Step 5 Compute the feedback gain from (23) and solve the Riccati differential equation from (28), respectively.
- Step 6 Compute the optimal control law from (22).
- Step 7 Solve the state equation from (29) and the costate equation from (21), respectively.
- Step 8 Evaluate the performance index from (30).

It is emphasized that the parameter estimation problem is settled from Step 2 to Step 4, while the optimal control problem is solved from Step 5 to Step 8. Moreover, the objective function used for parameter estimation is the sum of squared errors between the actual and estimated dynamic functions. This objective function offers a new insight into the proposed least-squares optimization for parameter estimation. In particular, we measure the mean squared error (MSE) as an estimate of estimation accuracy and determine the performance index using the control effort required to stabilize the dynamic model of the mangrove forest.

## 3. RESULTS AND DISCUSSION

Assume that the model parameter values given in Table 1 are unknown, and we only have the numerical solution of the model (1)-(3), which is the data-driven solution, as shown in Figure 1. Applying the Levenberg-Marquardt method discussed in the previous section, these model parameters are estimated, and their estimated values are shown in Table 2. The 99% confidence intervals for these parameter estimates are enclosed in the table. Notice that the parameter  $k$ , which is the environmental carrying capacity of forestry biomass, is fixed at 100 as mentioned in [11]. This parameter does not need to be estimated in our study, as it is not a variable but a constant.

Table 2. Parameter estimates and confidence intervals for mangrove forest model

| Parameter  | Original Value | Initial Value | Estimate Value | Lower Bound | Upper Bound |
|------------|----------------|---------------|----------------|-------------|-------------|
| $r$        | 1.5            | 1.35          | 1.4860         | 0.9355      | 1.9499      |
| $k$        | 100            | 100           | 100            | 63.9390     | 131.81      |
| $\beta$    | 0.9            | 1.03          | 0.8927         | 0.5668      | 1.2039      |
| $\gamma$   | 3.5            | 3.4           | 3.5373         | 2.2511      | 4.6497      |
| $e$        | 0.99           | 0.84          | 0.973          | 0.605       | 1.2772      |
| $d_1$      | 0.12           | 0.28          | 0.119          | 0.04079     | 0.2317      |
| $d_2$      | 0.1            | 0.15          | 0.1078         | 0.067322    | 0.1556      |
| $d_3$      | 0.785          | 0.98          | 0.7816         | 0.4956      | 1.1058      |
| $E$        | 3.8            | 3.9           | 3.8913         | 2.4886      | 5.1301      |
| $p$        | 0.1            | 0.15          | 0.1087         | 0.06732     | 0.1556      |
| $\alpha$   | 2.0            | 1.94          | 2.0230         | 1.2818      | 2.6485      |
| $\alpha_1$ | 1.5            | 1.3           | 1.4984         | 0.9189      | 1.9427      |
| $\alpha_2$ | 3.3            | 3.4           | 3.3102         | 2.2160      | 4.3870      |
| $\tau_1$   | 0.4            | 0.3           | 0.4004         | 0.2495      | 0.5261      |
| $\tau_2$   | 0.3            | 0.1           | 0.2915         | 0.09202     | 0.4066      |

The simulation result for the parameter estimates is shown in Table 3. The iteration number of 47 is taken to converge in 1.71 seconds and the MSE of  $9.7 \times 10^{-8}$  expresses the accuracy of the algorithm to estimate the model parameters. The graphical result of the model (1)-(3) is shown in Figure 2. Here, the solid lines represent the original population of trees and industrializations, while the dashed lines are regarded as the population of trees and industrializations when applying optimal parameter estimates. From the figure, it is clear that the trajectories of state estimates approximate closely the original state trajectories.

Table 3. Simulation result for parameter estimation in mangrove forest model

| Iteration Number | Mean Squared Error      | Elapsed Time (second) |
|------------------|-------------------------|-----------------------|
| 47               | $9.7016 \times 10^{-8}$ | 1.71451               |

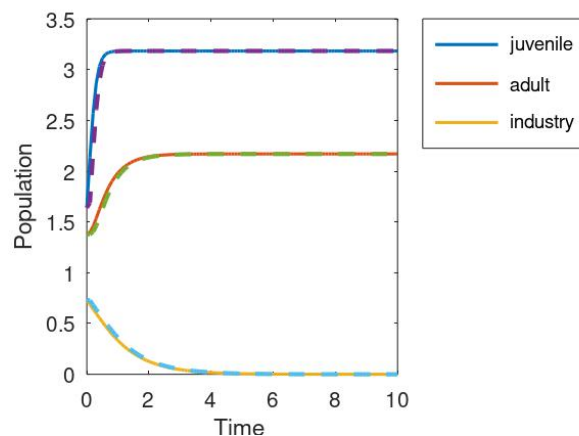


Figure 2. Population with parameter estimates in mangrove forest model

The accuracy of state estimates can be observed through the graphical result of state errors given by  $e_1$ ,  $e_2$  and  $e_3$ , as shown in Figure 3. These state errors that present the differences between the trajectories of state estimates and the original state trajectories are very close to zero after three years, indicating the accuracy of state estimates is highly expressed.

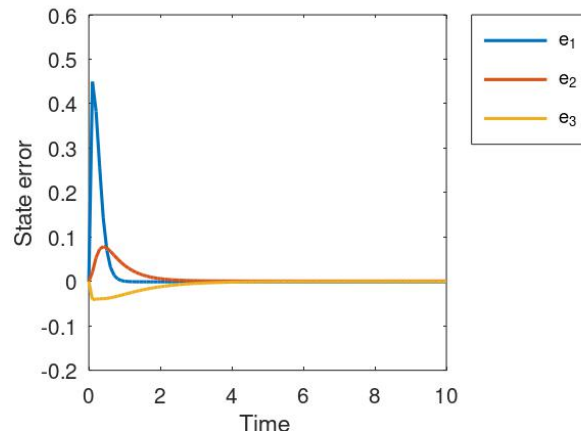


Figure 3. State error of mangrove forest model

Consider the initial states,  $x_1(0) = 1.6396$ ,  $x_2(0) = 1.3794$ ,  $x_3(0) = 0.7322$ , while the state transition matrix  $A$  and the control coefficient matrix  $B$  are given by

$$A = \begin{pmatrix} 0.8523 & 0 & 0 \\ 0.8686 & -1.4848 & 0 \\ 0 & -0.4236 & -0.6252 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -80 & 0 & 0 \\ 0 & -50 & 0 \\ 0 & 0 & 10 \end{pmatrix}.$$

Also, the following weighting matrices in the performance index

$$S(10) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad Q = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix} \quad \text{and} \quad R = \begin{pmatrix} 2.00 & 0 & 0 \\ 0 & 1.00 & 0 \\ 0 & 0 & 4.99 \end{pmatrix}$$

are taken. By applying the proposed computational algorithm, the optimal control law is designed to stabilize the state trajectories and the performance index is minimized.

Figure 4 shows the optimal state trajectories of the model (14) based on the model parameter estimates. The populations of juveniles and adults reach the steady state of 3.80 and 2.6, respectively, while industrialization stays at the steady state of  $1.7 \times 10^{-5}$ . This outcome indicates that the juvenile and adult populations are about 19.2% and 19.8% increments with having a control policy compared to the steady state of the original populations. Moreover, industrialization is under control by about a 63.8% reduction compared with the original industrialization steady state without control. With the optimal control policy, the optimal cost of adding the control input into the model (14) is 388.91 units.

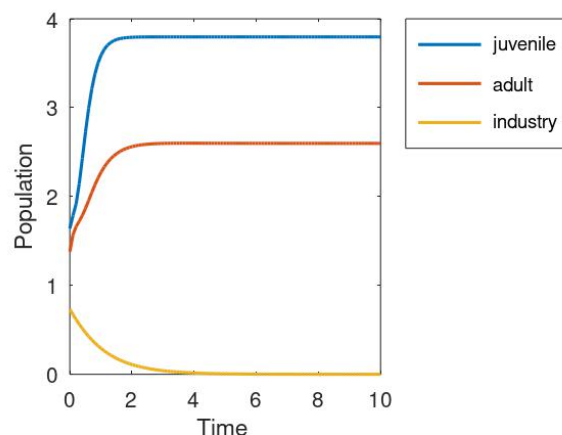


Figure 4. Optimal state trajectories of mangrove forest model

Figure 5 shows the optimal control laws  $u_1$ ,  $u_2$  and  $u_3$  for the model (14). To control the juvenile population, the control effort  $u_1$  is reduced from  $-0.1$  to  $-0.31$  in the first 0.4 years, then more control efforts are required to stabilize the growth rate of the juvenile population before the control effort stays around  $-0.0832$ . The population of adults has an initial control effort  $u_2$  at  $-0.38$ , and this control effort increases to  $0.125$ , but it reduces to  $-0.175$  later. After that, the control policy of the adult population increases gradually towards the value of  $-0.0088$  to maintain the adult population at its steady state. On the other hand, the control policy  $u_3$  decreases the development of industrialization, where the control effort increases exponentially from  $-0.54$  to  $-5.2 \times 10^{-5}$ . Therefore, these control policies regulate the populations of juveniles and adults by limiting the development of industrialization.

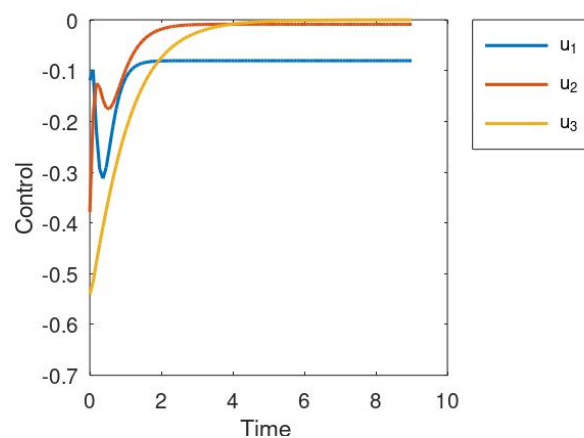


Figure 5. Optimal control trajectories of mangrove forest model

Figure 6 shows the optimal costate trajectories for the model (14) represented by  $p_1$ ,  $p_2$  and  $p_3$ . It is observed that the third costate trajectory reduces from  $0.27$  to  $2.4 \times 10^{-5}$ , indicating that the marginal cost is increased to ensure industrialization is under control. While the first and second costate trajectories stay around zero presenting that the steady state of the respective state trajectory can be achieved. Figure 7 shows that the stationary conditions given by  $H_{u_1}$ ,  $H_{u_2}$  and  $H_{u_3}$  stay along zero for which the optimal control trajectories are satisfied. Hence, by satisfying the stationary condition given in (16), we can confirm that the solutions for the model (14) are optimal.

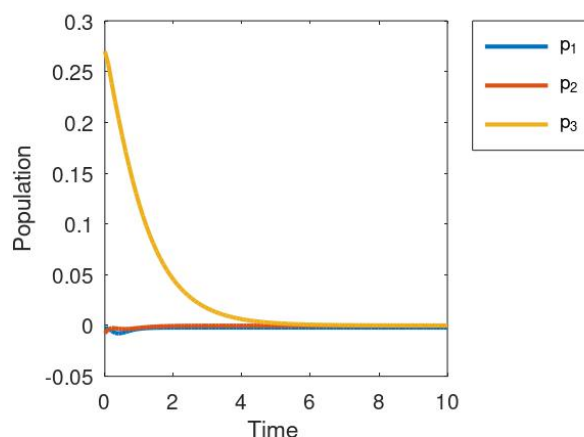


Figure 6. Optimal costate trajectories of mangrove forest model

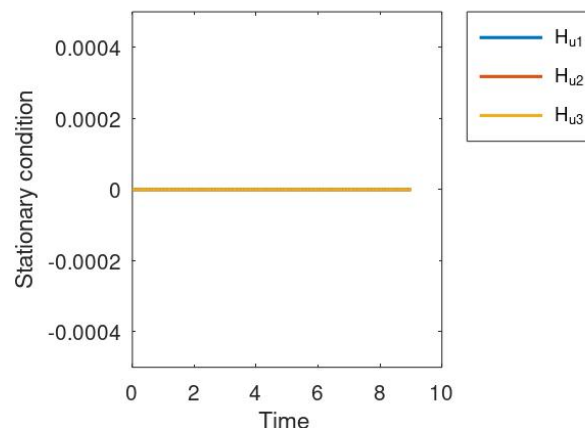


Figure 7. Stationary conditions of mangrove forest model

#### 4. CONCLUSION

The paper presented an efficient computational algorithm that integrates parameter estimation and optimal control for the sustainable management of mangrove forests. A least-squares optimization problem was introduced, and the Levenberg-Marquardt method was applied to estimate the model parameters. Later, an optimal control problem with the model parameter estimates was formed to derive cost-effective management policies that promote sustainable mangrove conservation. The simulation results for the illustrative example demonstrated the accuracy of the estimate and the efficiency of the control policy. Hence, our approach demonstrates significant usefulness in mangrove forest management and will have practical implications for the mathematical modelling of mangrove forest dynamics.

While our model purposefully isolates the core dynamics between juvenile and adult tree populations alongside the industrialization index, it omits external ecological drivers such as climate dynamics. Introducing these factors would significantly complicate the system's dimensionality and parameter estimation. Consequently, maintaining a parsimonious ODE structure ensures model tractability, though integrating climate variables remains a crucial avenue for future work as more comprehensive environmental datasets become available. In future research, it is recommended that the control decision on more complicated dynamic models be explored using the proposed approach.

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## BIOGRAPHIES OF AUTHORS



**Nurul Eliana Atika Mohd Idrus** received the B.Sc. degree in mathematics technology from the Universiti Tun Hussein Onn Malaysia in 2020. She is currently a master's student in the Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia (Pagoh Campus). Her research interests include ordinary differential equations, numerical methods and operational research. She can be contacted at email: [elianaidrus97@gmail.com](mailto:elianaidrus97@gmail.com)



**Sie Long Kek** ORCID · Google Scholar · Scopus · Web of Science received the M.Sc. and Ph.D. degrees in applied mathematics from the Universiti Teknologi Malaysia, in 2001 and 2011, respectively. He is currently a senior lecturer in the Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia (Pagoh Campus). His research interests include optimization and control, operational research and management science, modelling and simulation, parameter estimation, Kalman filtering and computational mathematics. He has published over 50 papers in refereed journals and six book chapters. He is a reviewer for peer-reviewed research journals. He has joined the Technical Program Committee for over 12 international conferences. He can be contacted at email: [slkek@uthm.edu.my](mailto:slkek@uthm.edu.my).