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Significance of Weighted-Type Fractional Fourier Transform in FIR Filters

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Abstract

The desired frequency response of a filter is periodic in frequency and can be expanded in Fourier series. One possible way of obtaining FIR filter is to truncate the infinite Fourier series. But abrupt truncation of the Fourier series results in oscillation in the pass band and stop band. These oscillations are due to slow convergence of the Fourier series by the Gibb's phenomenon. To reduce these oscillations the Fourier coefficients of the filter are modified by multiplying the infinite impulse response with a finite weighing sequence called a window. The Fourier transform (FT) of a window consists of a central lobe and side lobes. The central lobe contains most of the energy of the window. To get an FIR filter, the desired impulse response and window function are multiplied, which results to give finite length non-causal sequence. Since Fractional Fourier Transform (FrFT) is generalization of FT. Here an attempt is to implement filters using window by using Weighted Type Fractional Fourier Transform (WFrFt), differentiator and integrator using weighted FrFt is also present.

***Keywords**: Fourier Transform, FIR Filter, frequency response, Gibb's phenomenon, differentiator, integrator*

1. Introduction

Fourier transform has proved to be a very useful tool for application in many disciplines such as quantum mechanics, signal processing etc. Availability of Discrete Fourier Transform (DFT) [1-4] gives an excellent representation of a continuous transform. Analysis of signals using tools of Fourier series and Fourier Transform (FT) have been discussed in [12]. Window functions are weighted in time domain, are applied to the signal under consideration. The process of convoluting the measured signal with a smoothly ending window function is called the windowing technique. Windowing is done to make an infinitely long function, finite in length so that the frequency content of signal of interest can be measured .The resultant truncated signal exhibits various spectral characteristics. There are few important parameters in spectral analysis of function [5], which are [I] Half Main Lobe Width (HMLW) or Band Width (BW) is defined as the frequency at which the main lobe drops to peak ripple value of the side lobes [II] Maximum Side Lobe Level (MSLL) or Side Lobe Attenuation (SLA) is the largest side lobe level relative to the main lobe peak gain [III] Side Lobe Fall-off Ratio (SLFOR) or Ripple Ratio (RR) is the asymptotic decay rate of side lobe level in decibels per the decade of frequency of the peaks of side lobes.

An ideal filter should posses linear phase, ideal filters are unstable because the sinc function is not sum able. Methods of truncating need to be used. A Triangular window may be regarded as the convolution of two rectangular windows. Windowing is a process of multiplying impulse response by a consider window function in the time domain is equivalent to periodic convolution of filter spectrum and window spectrum in frequency domain. One of the best ways to reduce oscillations and over shoots is by using a triangle window it helps side lobe decay faster. Spectrum of any window should have a narrow main lobe and small side lobe levels. For a given window length, it is not possible to minimize both main lobe width and side lobe levels simultaneously. Design of window requires tradeoff between these two conflicting requirements.

Window functions provide more smoothing through convolution operation in the time domain, as a result, transition region is wider in Finite Impulse Response (FIR) filter. To reduce the width of this transition region, we can simply increase the length of the window, resulting in a larger filter. This trade off improves by analyzing these filters with FrFT. The fractional Fourier transform (FRFT) is a powerful time-frequency mathematical tool, which has widely applications in quantum mechanics, signal processing, optics, communications, etc. The Fourier transform

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(FT) has four eigenvalues: 1, j, −1 and−j, while the FRFT generalizes the FT in the way of eigen values fractionalization. The different fractional schemes lead to a variety of definitions of the FRFT which consist of two mainly types: (i) the chirp-type FRFT (CFRFT) and (ii) the weighted-type FRFT (WFRFT) [10].

J. Harris analyses signals using different types of windows in 1978 [11], in which he explained the figure of merit of different windows based on MSLL, SLFOR and HMLW. It is based on Fourier Transform (FT), Fast Fourier transform (FFT). Hence, analysis of signals is completely based on characteristics of window functions being used. The window chosen for truncating the infinite impulse response should have some desirable characteristics. The central lobe of frequency response of the window function should contain most of the energy and should be narrow. The first side lobe level of the frequency response should be small and the side lobes of frequency response decrease in energy, rapidly as ‘w’ → 'π'.

2. Weighted FrFT

Fractional Fourier Transform (WFrFT) is a generalization of the Fourier Transform [6] proposed some years ago by many authors. The fractional Fourier Transform of a function x, with an angleα, is defined as [10]

$$F^{\alpha}(x(n)) = w0.x(t) + w1.x(u) + w2.x(-t) + w3.x(-u) \quad (1)$$

$$\text{Weight matrix (p)} = \cos\left[\frac{(\alpha-p)\Pi}{4}\right]\cos\left[\frac{2(\alpha-p)\Pi}{4}\right]\exp\left[\frac{j3(\alpha-p)\Pi}{4}\right]$$

Where p=0, 1, 2, 3 (2)

x(t)=function in time domain
x(u)=frequency domain of x(t)
x(-t)=inverse time domain of x(t)
x(-u)=inverse frequency domain of x(u)

FrFT with $\alpha = \pi/2$ corresponds to the classical Fourier transform, and one with $\alpha = 0$ corresponds to the identity operator. FrFT order can also be used in place of FrFT angle. The relationship between FrFT order and angle is given by $\alpha = a\pi/2$. In signal processing, FrFT has been applied to optimal Wiener filtering and matched filtering.

3. Modified Weighted FrFT

As per the literature survey the WFrFT values are almost equal to FFT at a=1.So our aim is to attain values more than FFT values.

In view of the above facts, intensive investigations are carried out in the present work for implementation of Weighted Fractional Fourier Transform in digital filter design. New differentiator and integrator will be design based on WFrFT.

$$F^{\alpha}\big(x(n)\big) = w0.x(t).k0 + w1.x(u).k1 + w2.x(-t).k2 + w4.x(-u).k3 \quad (3)$$

Where k0, k1, K2, k3 are the proposed constants, for different values of these constants the spectral characteristics of window functions will give better values than FT, WFrFT.

Comparison of spectral parameters of different window functions with FT, WFrFT, and Modified WFrFT

S.NO	TYPE OF DOMAIN	TYPEOF WINDOW	SLA(dB)
1	FT	RECTANGLE	-13.2
		HAMMING	-41.2
		HANNING	-31.5
2	WFrFT	RECTANGLE	-10.6
		HAMMING	-33.0
		HANNING	-26.7
3	MODIFIED WFrFT	RECTANGLE	-15.9
		HAMMING	-44.3
		HANNING	-34.2

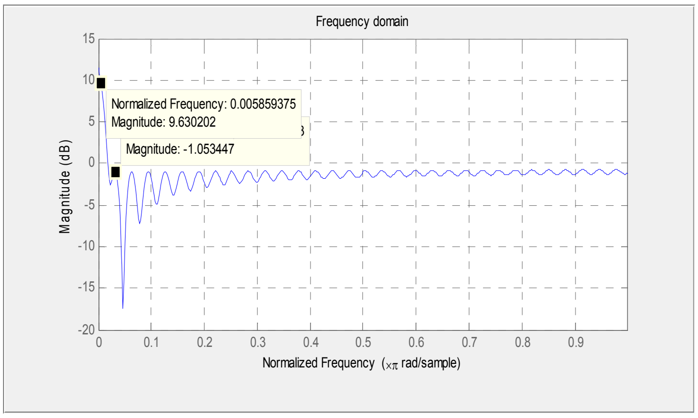


Figure 1. Spectral characteristics of rectangle window with wfrft

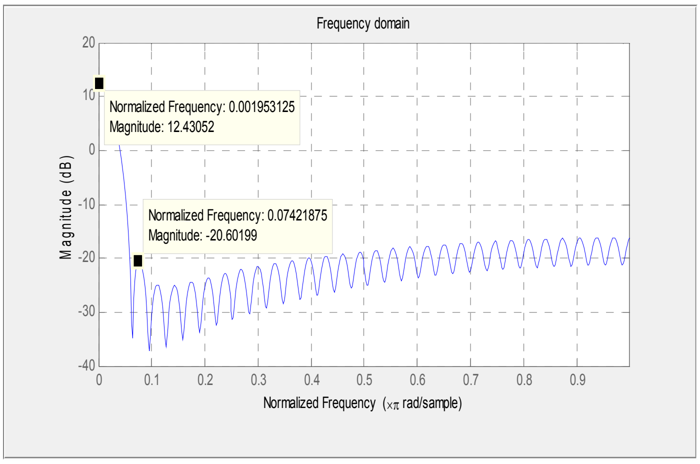


Figure 2. Spectral characteristics of Hamming window with wfrft

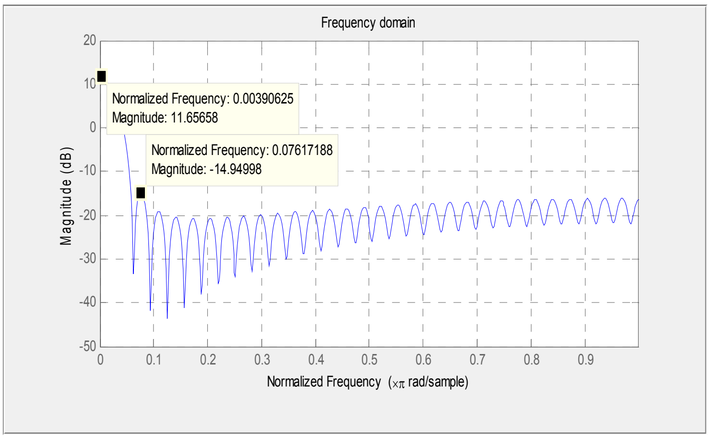


Figure 3. Spectral characteristics of hanning window with wfrf

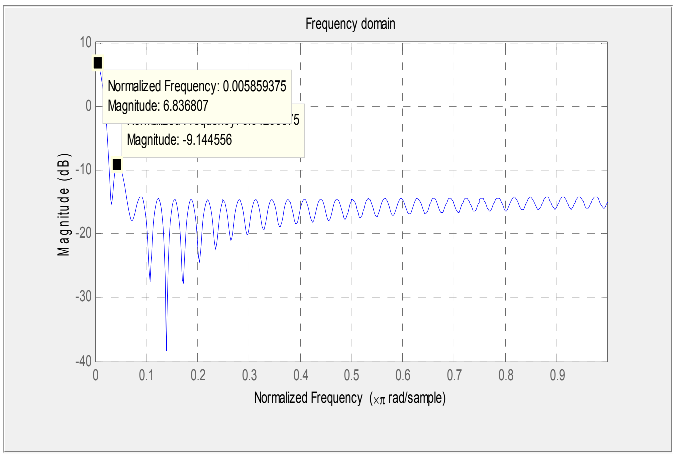


Figure 4. Spectral characteristics of rectangle window with modified wfrft

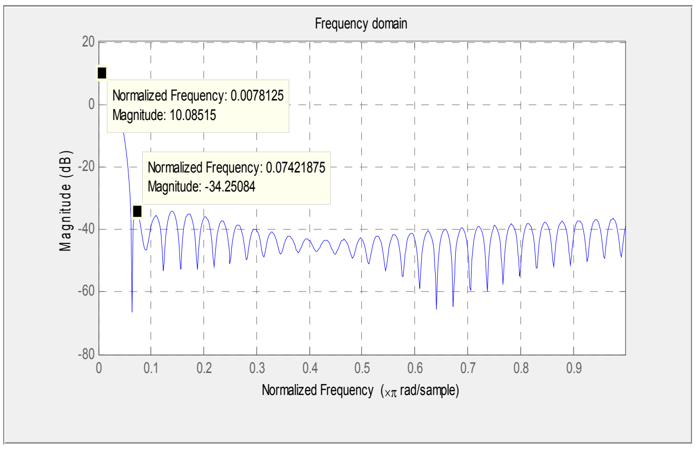


Figure 5. Spectral characteristics of hamming window with modified wfrft

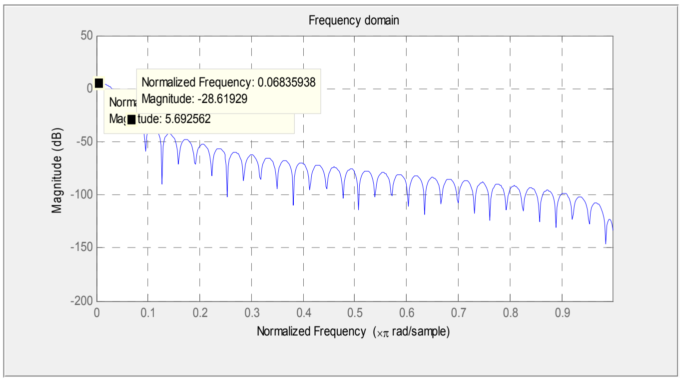


Figure 6. Spectral characteristics of hanning window with modified wfrft

4. FIR Filter Design Techniques

Various types of FIR filter design techniques which approximately produce the ideal response are:

- Numerical analysis method
- Window method

4.1 Proposed Scheme

We made a attempt to achieve FIR filter by introducing weighted frft to filter as given below formula [7, 8]

$$H_d(w).F^{\alpha}[W(n)] \tag{4}$$

$H_d(w)$=Desired infinite frequency response of a filter
$F^{\alpha}[W(n)]$=modified weighted fractional Fourier transform of a window function
Desired frequency response of infinite impulse response

$$h_d(n) = \frac{w_c}{\Pi} \quad \text{for } n = \alpha$$

$$= \frac{\sin w_c(n-\alpha)}{\Pi(n-\alpha)} \quad \text{for } n \neq \alpha \tag{5}$$

Where $\alpha = \dfrac{N-1}{2}$

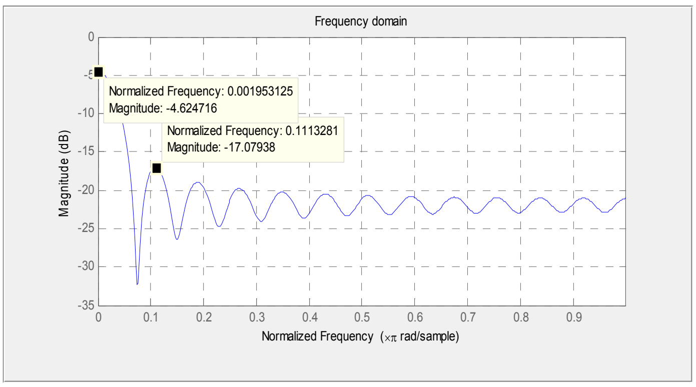


Figure 7. The frequency response of low pass filter for rectangle window using modified WFRFT

5. Differentiator and Integrator

The frequency response of an ideal digital differentiator is linearly proportional to frequency. It is given by

$H_d(e^{jw)} = jw \quad -\Pi \leq w \leq \Pi$

Where $\alpha = (N-1)/2$ (6)

The ideal impulse response of a digital differentiator with linear phase is given by for N even

$$h_d(n) = -\frac{\sin(n-\alpha)\Pi}{\Pi(n-\alpha)^2} \quad \text{for } n \neq \alpha$$

$$=0 \quad \text{for } n = \alpha \quad (7)$$

The finite impulse response can be obtained by truncating $h_d(n)$ by using a window with modified WFrFT.

Then h (n) = $h_d(n)$ .w(n) (8)

The impulse response of differentiator is obtained by direct truncation and by using a hamming window.

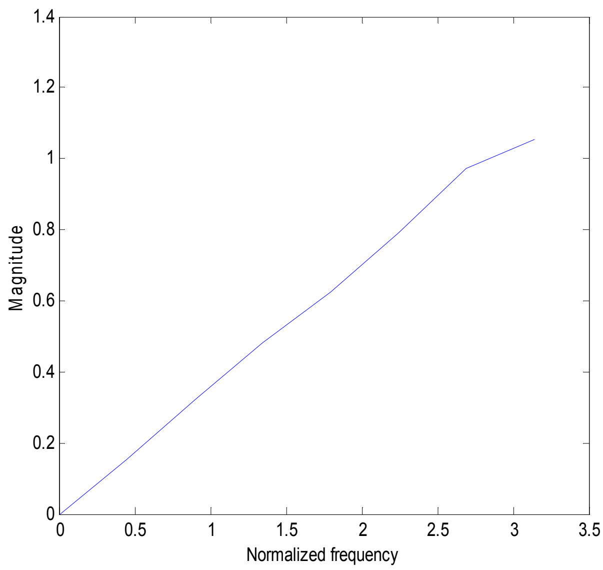
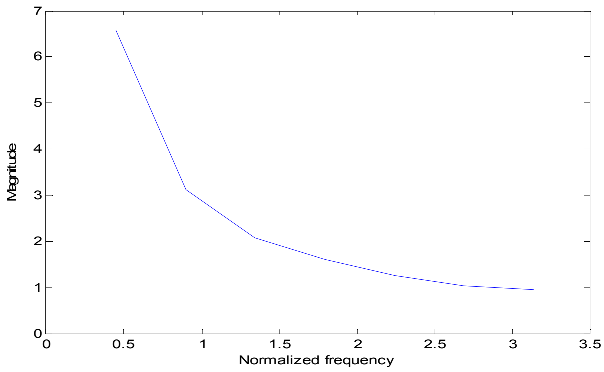


Figure 8. The impulse response of differentiator with modified wfrft of hamming window i
Since we know that differentiators and integrators are inverse each other the transfer function of integrator is 1/ $h_d(n)$



The frequency response of integrator with modified wfrft is shown in figure 9

6. Conclusion

By observing the WFrFT mathematical definition, the analysis of all windows (rectangle, hamming and hanning) is presented. And also the drawback of WFrFT is also solved by comparing WFrFT and MODIFIED WFrFT spectral characteristics of windows. Finally Differentiator and Integrator are also observed.

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